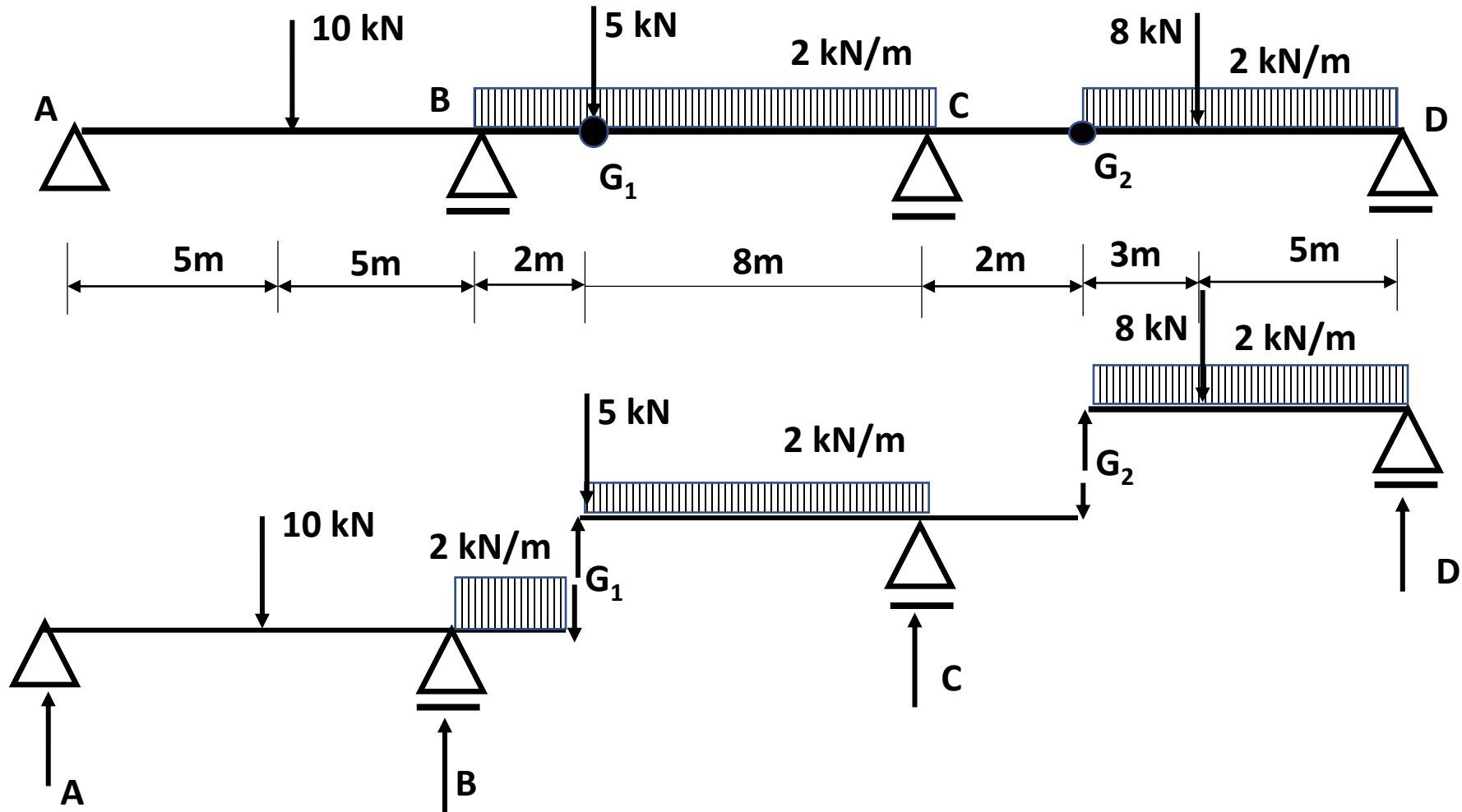


ÖRNEK PROBLEMLER

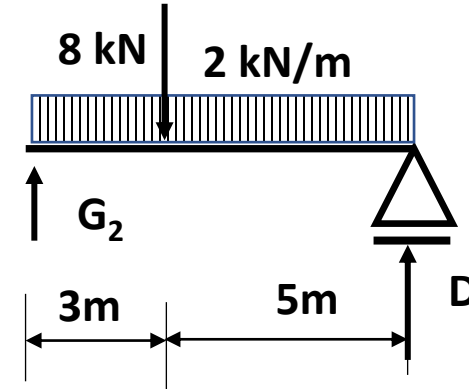
Gerber Kirişleri ve Çerçeveler

ÖRNEK 1



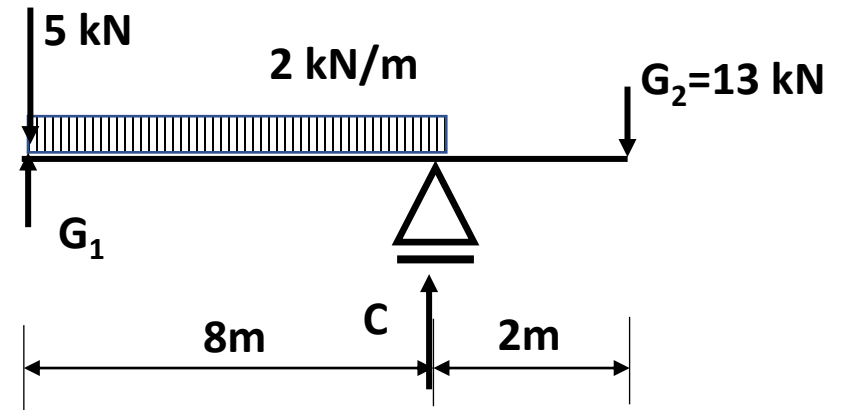
$$\sum M_D = 0 \quad G_2 = \frac{2 \cdot 4 \cdot 8}{8} + \frac{8 \cdot 5}{8} = 13 \text{ kN}$$

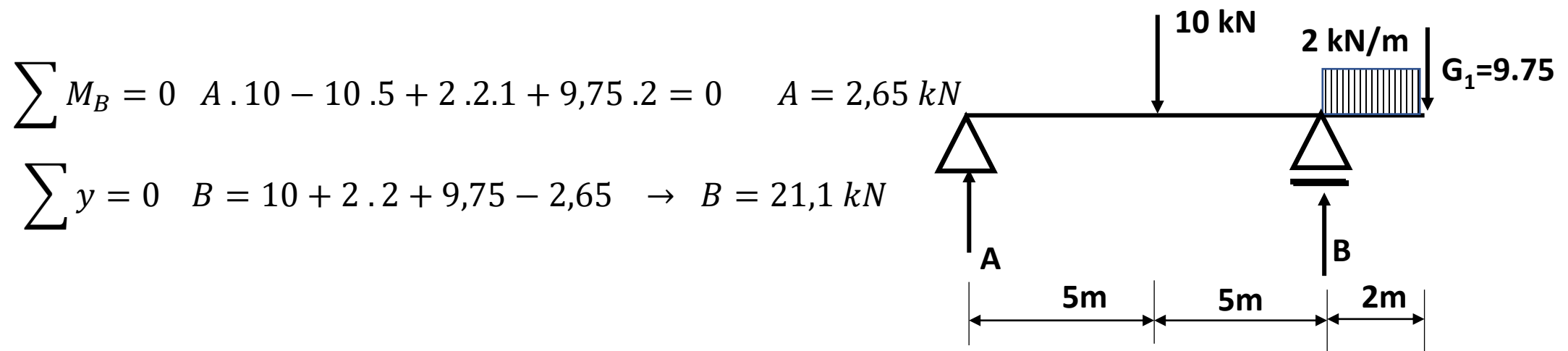
$$\sum y = 0 \quad D = -13 + 16 + 8 = 11 \text{ kN}$$

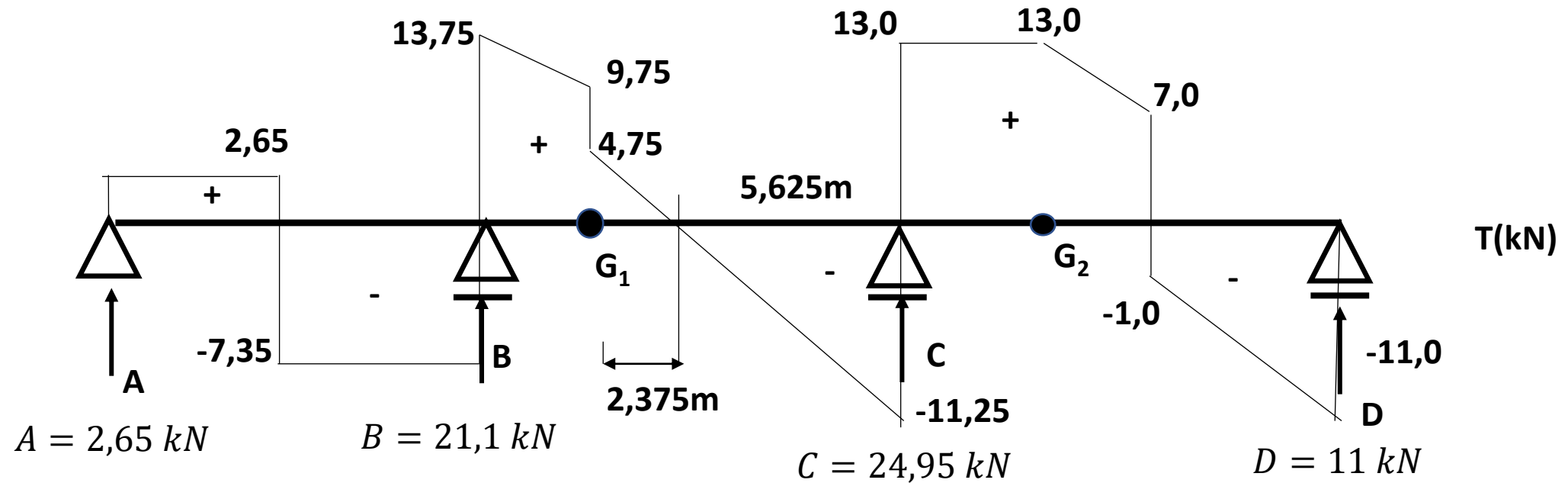
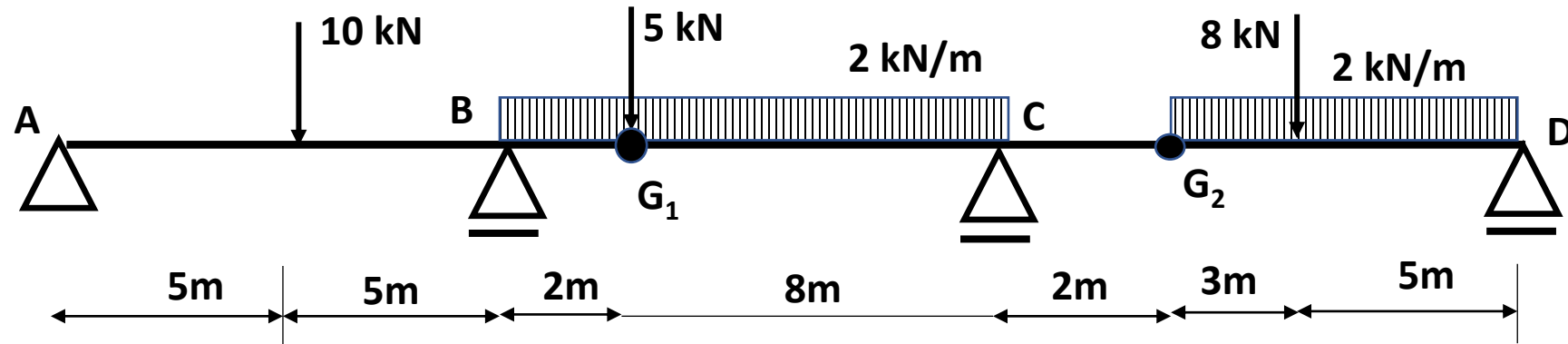


$$\sum M_{G1} = 0 \quad -C \cdot 8 + 13 \cdot 10 + 2 \cdot 8 \cdot 4 = 0 \quad C = 24,95 \text{ kN}$$

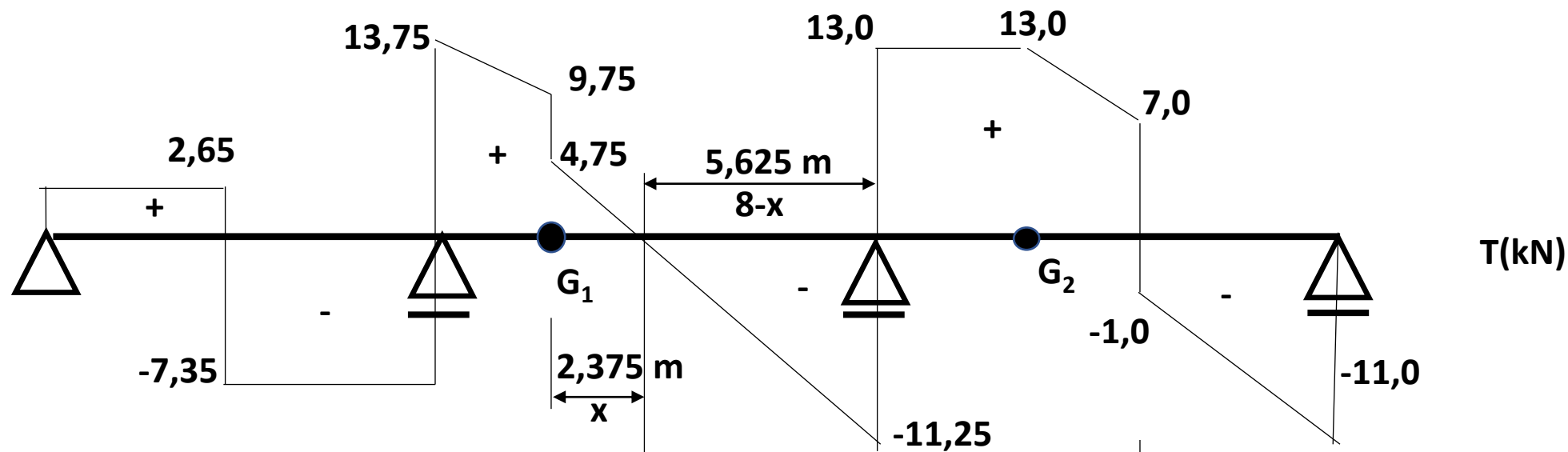
$$\sum y = 0 \quad G_1 = 5 + 8 \cdot 2 + 13 - 24,95 \rightarrow G_1 = 9,75$$



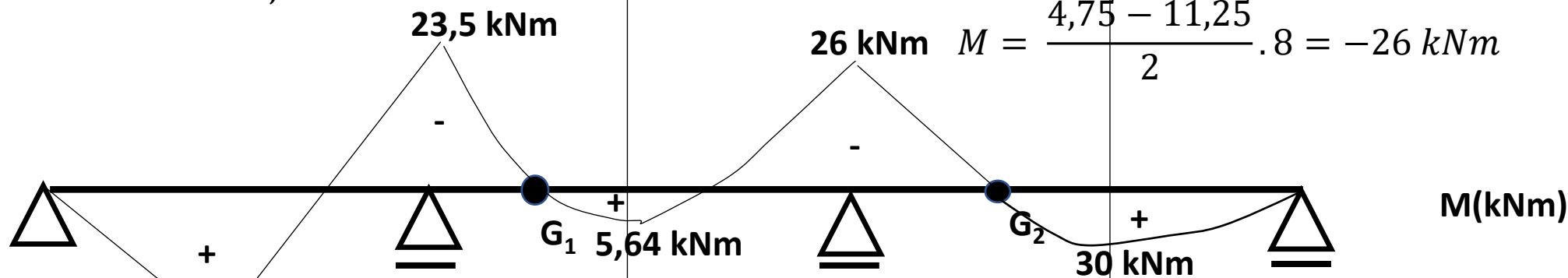




$$\frac{4,75}{x} = \frac{11,25}{8-x} \rightarrow x = 2,375 \text{ m}$$



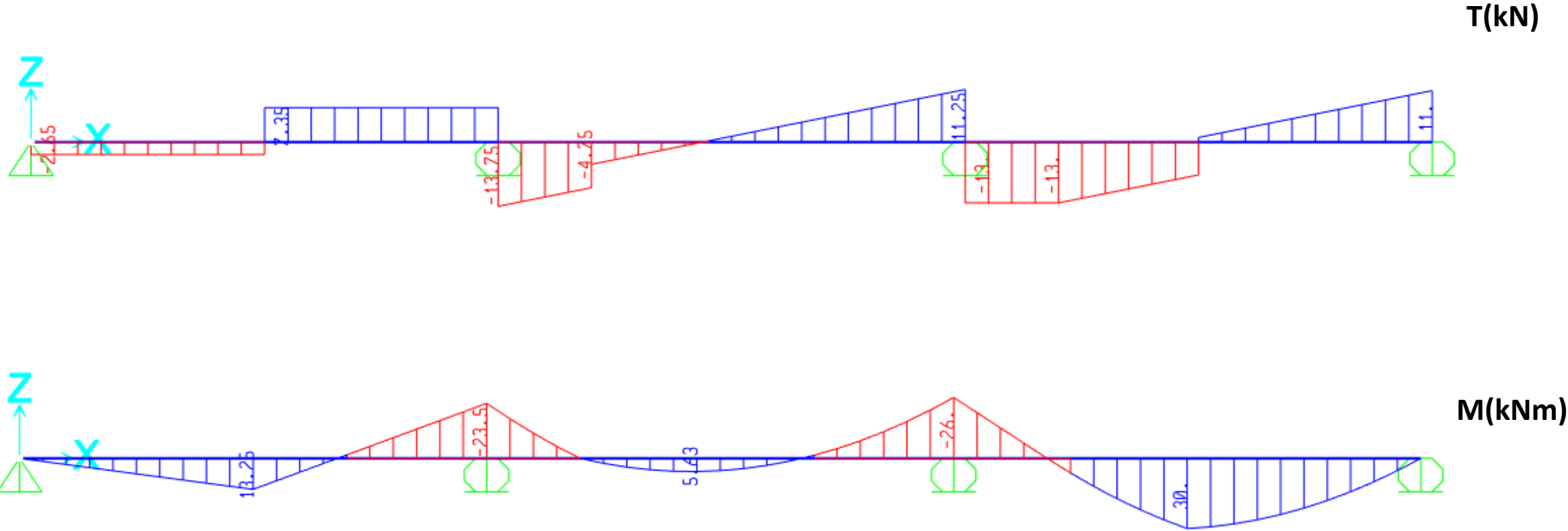
$$M = 13,25 - 7,35 \cdot 5 = -23,5 \text{ kNm}$$



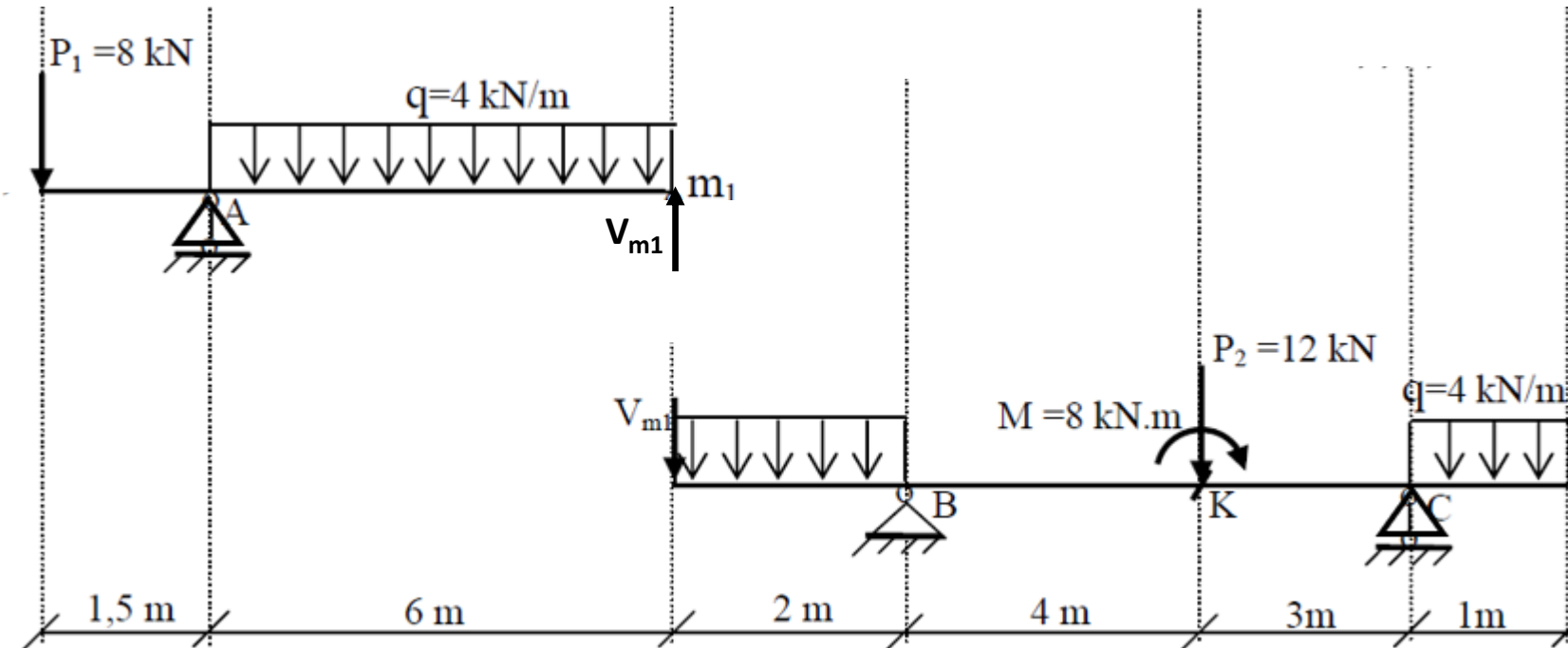
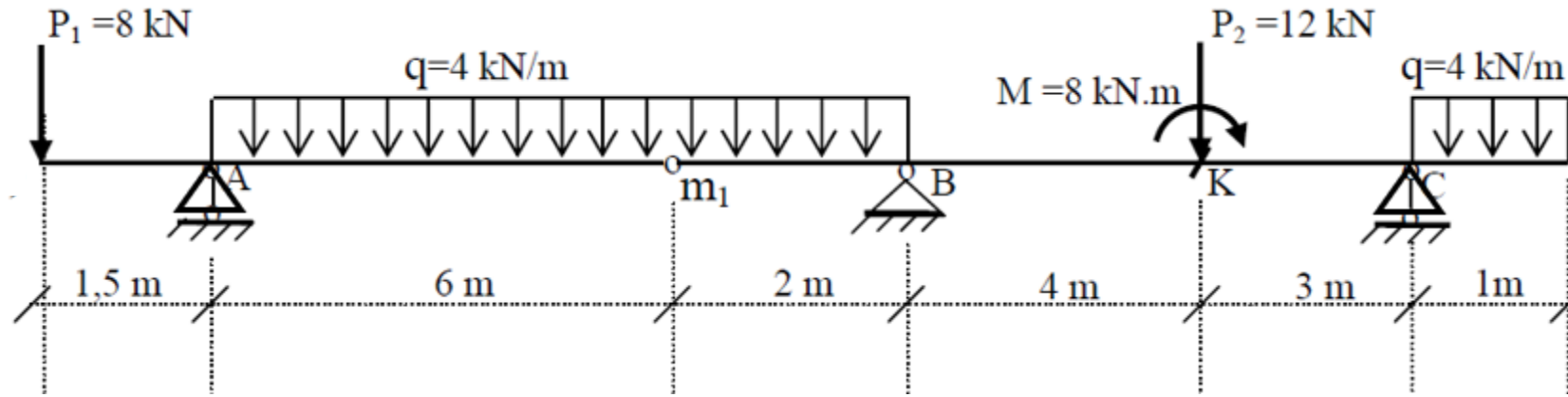
$$M = 2,65 \cdot 5 = 13,25 \text{ kNm}$$

$$M_{max} = \frac{4,75 \cdot 2,375}{2} = 5,64 \text{ kNm}$$

$$M_{max} = \frac{13 + 7}{2} \cdot 3 = 30 \text{ kNm}$$



ÖRNEK 2



Mesnet tepkileri:

$$\sum M_{m_1} = -P_1 \cdot (7,5) + V_A \cdot 6 - q \cdot 6 \cdot 3 = 0 \Rightarrow$$

$$V_A = \frac{8 \cdot (7,5) + 4 \cdot 6 \cdot 3}{6} = 22 \text{ kN}$$

$$\sum M_A = -V_{m1} \cdot 6 + q \cdot 6 \cdot 3 - P_1 \cdot (1,5) = 0 \Rightarrow$$

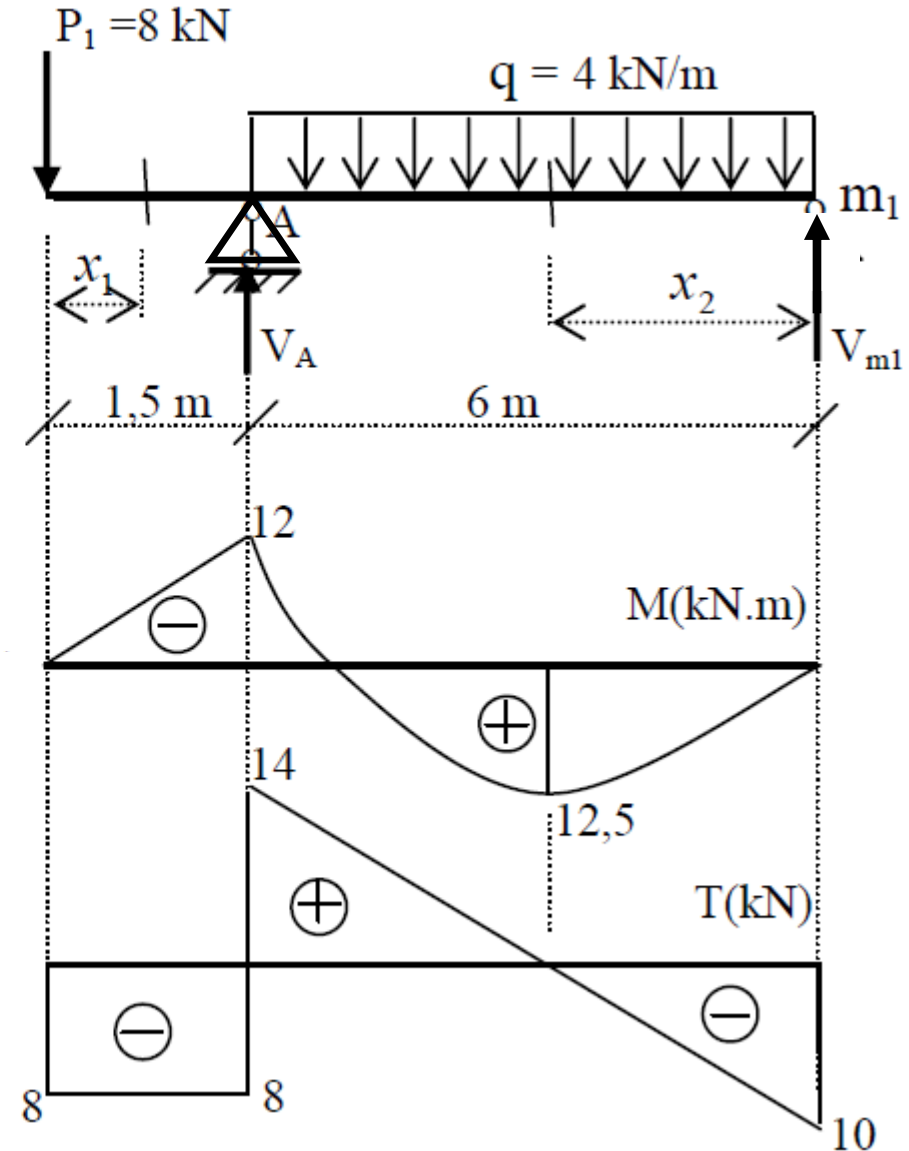
$$V_{m1} = \frac{4 \cdot 6 \cdot 3 - 8 \cdot (1,5)}{6} = 10 \text{ kN}$$

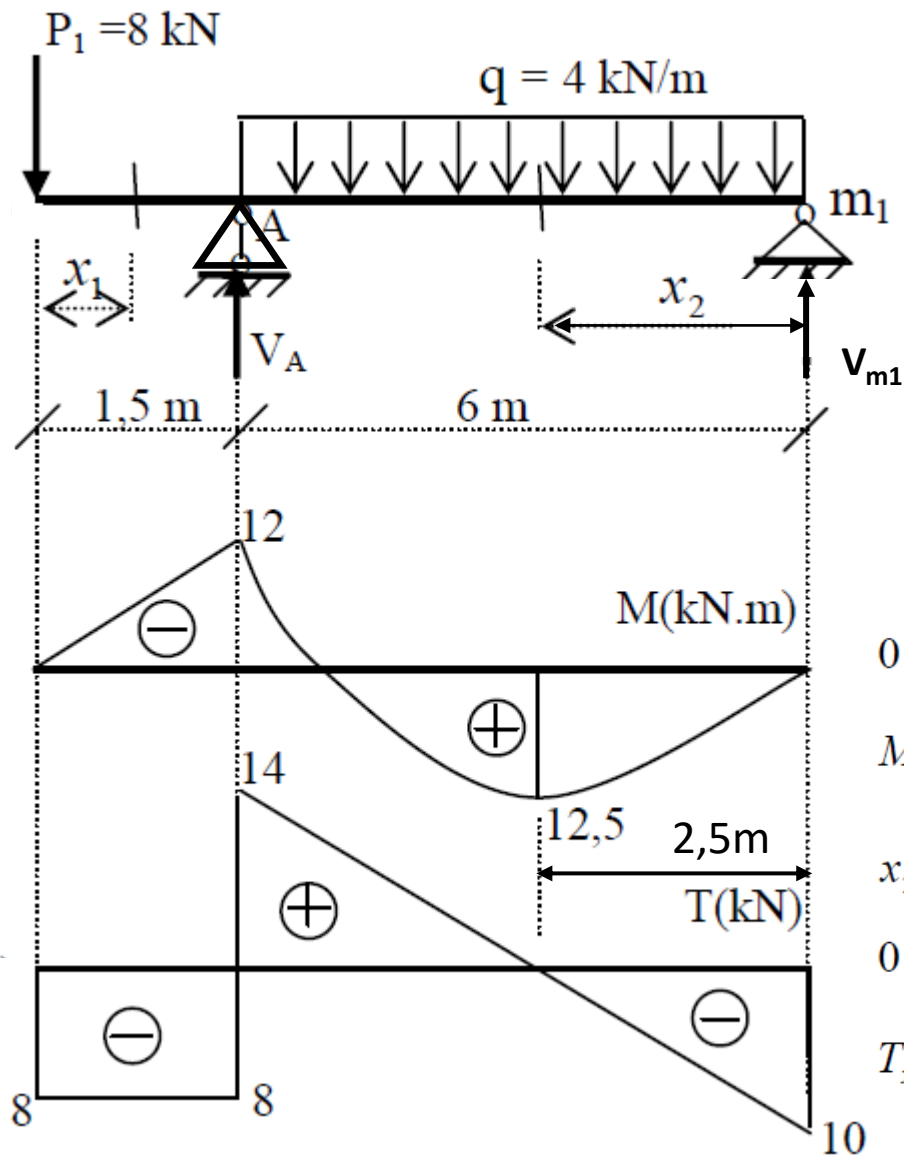
Sağlama :

$$\sum Y = -P_1 + V_A - q \cdot 6 + V_{m1} = 0 \Rightarrow$$

$$-8 + 22 - 4 \cdot 6 + 10 = 0.$$

Her bölge için **M ve T**
değerlerinin hesaplanması:





$$0 \leq x_1 \leq 1,5$$

$$M_{x1} = -P_1 \cdot x_1; \quad T_1 = -P_1 = 8 \text{ kN}.$$

$$x_1 = 0 \Rightarrow M_{x1} = 0; \quad x_1 = 1,5 \text{ m} \Rightarrow M_{x1} = -12 \text{ kN.m}.$$

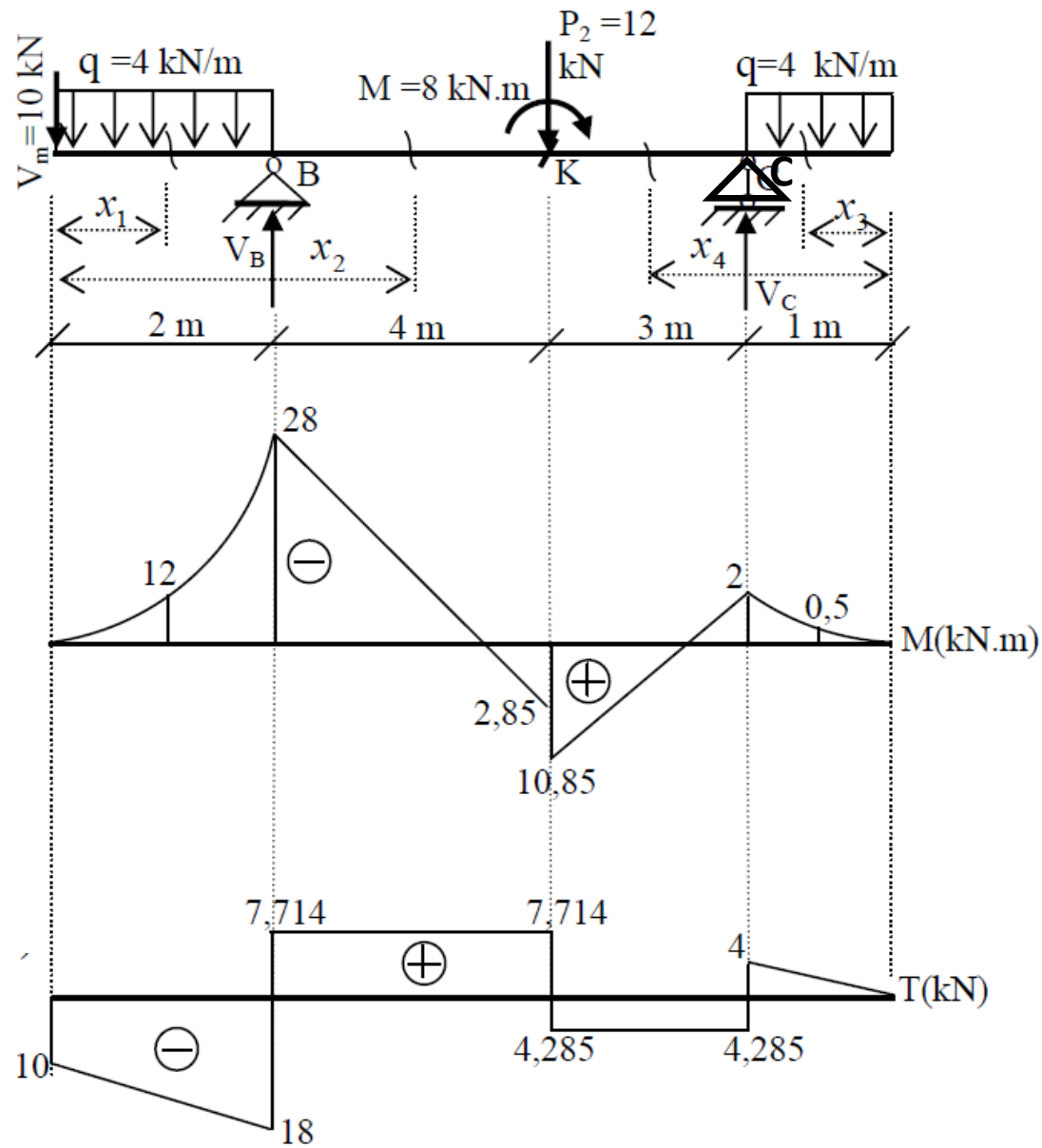
$$0 \leq x_2 \leq 6$$

$$M_{x2} = V_{m1} \cdot x_2 - q \cdot \frac{x_2^2}{2}; \quad T_{x2} = -V_{m1} + q \cdot x_2 \quad x_2 = 0 \rightarrow M_{x2} = 0, T_{x2} = -10 \text{ kN}$$

$$x_2 = 6 \text{ m} \Rightarrow M_{x2} = 10 \cdot 6 - 4 \cdot \frac{36}{2} = -12 \text{ kN.m}, \quad T_{x2} = -10 + 4 \cdot 6 = -14 \text{ kN}.$$

$0 \leq x_2 \leq 6$ aralığında eğilme momentinin maksimum değer aldığı kesitin x_0 uzaklığı:

$$T_{x0} = -10 + q \cdot x_0 = 0 \Rightarrow x_0 = \frac{10}{4} = 2,5 \text{ m}. \quad M_{\max} = 10 \cdot (2,5) - 4 \cdot \frac{(2,5)^2}{2} = 12,5 \text{ kN.m}$$



$$V_A = 22 \text{ kN} \quad V_{m1} = 10 \text{ kN}$$

Mesnet tepkileri:

$$\sum M_C = -V_{m1} \cdot 9 - q \cdot 2.8 + V_B \cdot 7 - P \cdot 3 + M + q \cdot 1 \cdot (0.5) = 0 \Rightarrow$$

$$V_B = \frac{10.9 + 4.2.8 + 12.3}{7} - \frac{8 + 4.1 \cdot (0.5)}{7} = 25.7143 \text{ kN.}$$

$$\sum M_B = q \cdot 1 \cdot (7.5) - V_C \cdot 7 + P \cdot 4 + M - V_{m1} \cdot 2 - q \cdot 2 \cdot 1 = 0 \Rightarrow$$

$$V_C = \frac{4.1 \cdot (7.5) + 12.4 + 8 - 10.2}{7} - \frac{4.2 \cdot 1}{7} = 8.2857 \text{ kN.}$$

Kontrol

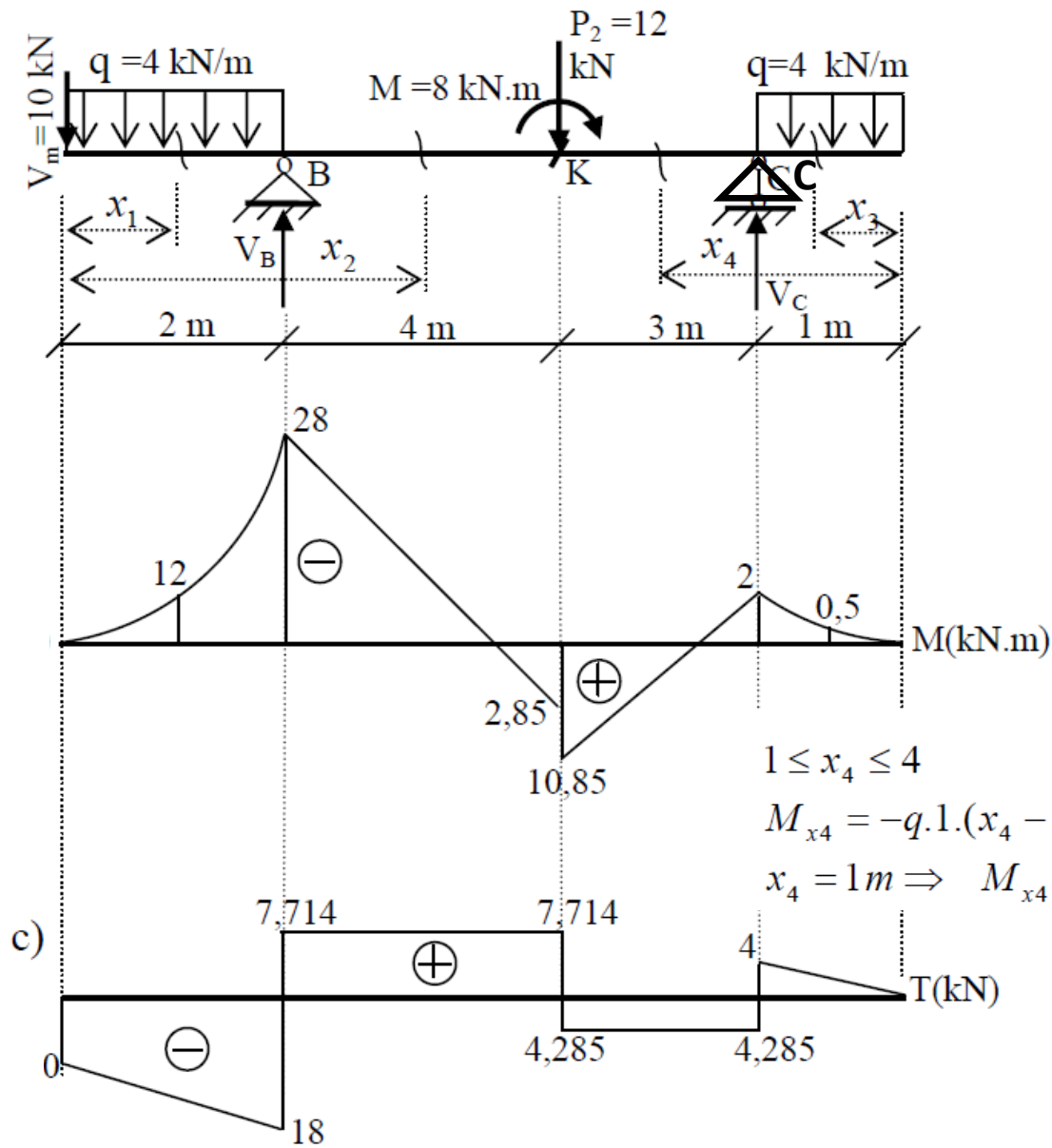
$$\sum Y = -V_{m1} - q \cdot 2 + V_B - P + V_C - q \cdot 1 = 0 \Rightarrow -10 - 4.2 + 25.7143 - 12 + 8.2857 - 4.1 = 0$$

$$0 \leq x_1 \leq 2$$

$$M_{x1} = -V_{m1} \cdot x_1 - q \cdot \frac{x_1^2}{2}; \quad T_{x1} = -V_{m1} - q \cdot x_1. \quad x_1 = 0 \Rightarrow M_{x1} = 0, T_{x1} = -10 \text{ kN,}$$

$$x_1 = 1 \text{ m} \Rightarrow M_{x1} = -12 \text{ kN.m,}$$

$$x_1 = 2 \text{ m} \Rightarrow M_{x1} = -10.2 - 4 \cdot \frac{2^2}{2} = -28 \text{ kN.m}; \quad T_{x1} = -10 - 4.2 = -18 \text{ kN.}$$



$$2 \leq x_2 \leq 6 \quad V_B = 25,7143 \text{ kN} \quad V_C = 8,2857 \text{ kN} \quad V_{m1} = 10 \text{ kN}$$

$$M_{x_2} = -V_{m1} \cdot x_2 - q \cdot 2 \cdot (x_2 - 1) + V_B \cdot (x_2 - 2);$$

$$T = -V_{m1} - q \cdot 2 + V_B \Rightarrow -10 - 4 \cdot 2 + 25,7143 = 7,7143 \text{ kN}.$$

$$x_2 = 2 \text{ m} \Rightarrow M_{x_2} = -10 \cdot 2 - 4 \cdot 2 \cdot 1 = -28 \text{ kN.m},$$

$$x_2 = 6 \text{ m} \Rightarrow M_{x_2} = -10 \cdot 6 - 4 \cdot 2 \cdot 5 + (25,7143) \cdot 4 = 2,857 \text{ kN.m}.$$

$$0 \leq x_3 \leq 1$$

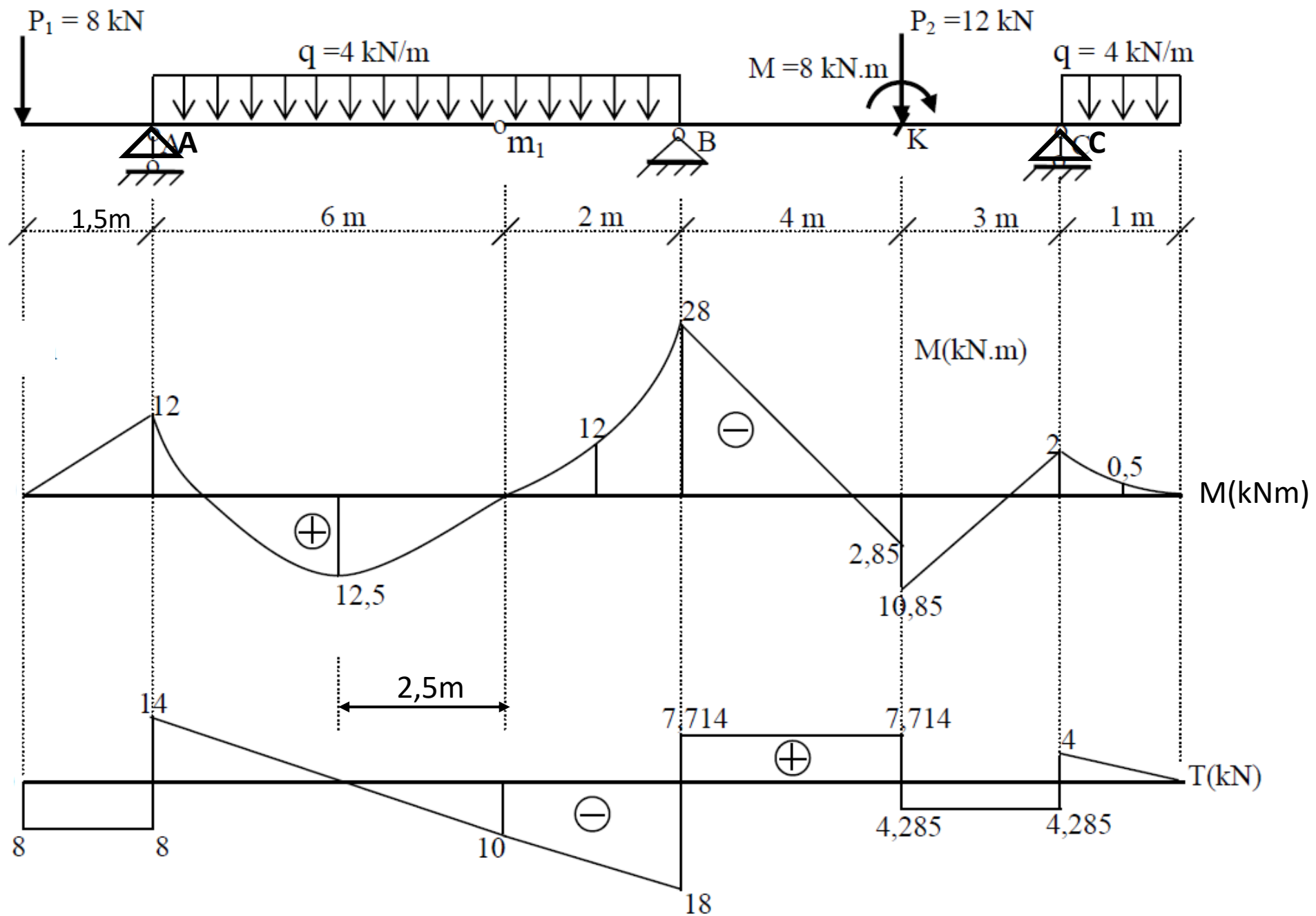
$$M_{x_3} = -q \cdot \frac{x_3^2}{2}; \quad T_{x_3} = q \cdot x_3. \quad x_3 = 0 \Rightarrow M_{x_3} = 0; \quad T_{x_3} = 0. \quad x_3 = 0,5 \text{ m} \Rightarrow$$

$$M_{x_3} = -0,5 \text{ kN.m}. \quad x_3 = 1 \text{ m} \Rightarrow M_{x_3} = -4 \cdot \frac{1^2}{2} = -2 \text{ kN.m}, \quad T_{x_3} = 4 \cdot 1 = 4 \text{ kN}.$$

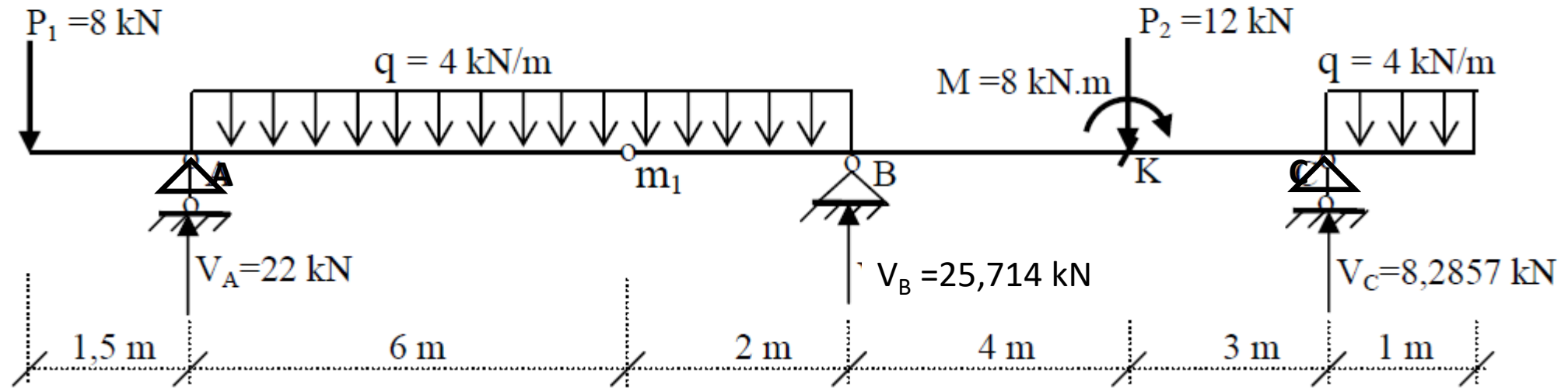
$$1 \leq x_4 \leq 4$$

$$M_{x_4} = -q \cdot 1 \cdot (x_4 - 0,5) + V_C \cdot (x_4 - 1); \quad T = q \cdot 1 - V_C = 4 \cdot 1 - 8,2857 = -4,2857 \text{ kN}.$$

$$x_4 = 1 \text{ m} \Rightarrow M_{x_4} = -2 \text{ kN.m}, \quad x_4 = 4 \text{ m} \Rightarrow M_{x_4} = -4 \cdot 1 \cdot (3,5) + (8,2857) \cdot 3 = 10,8571 \text{ kN.m}.$$



Kirişin tümünün denge kontrolü

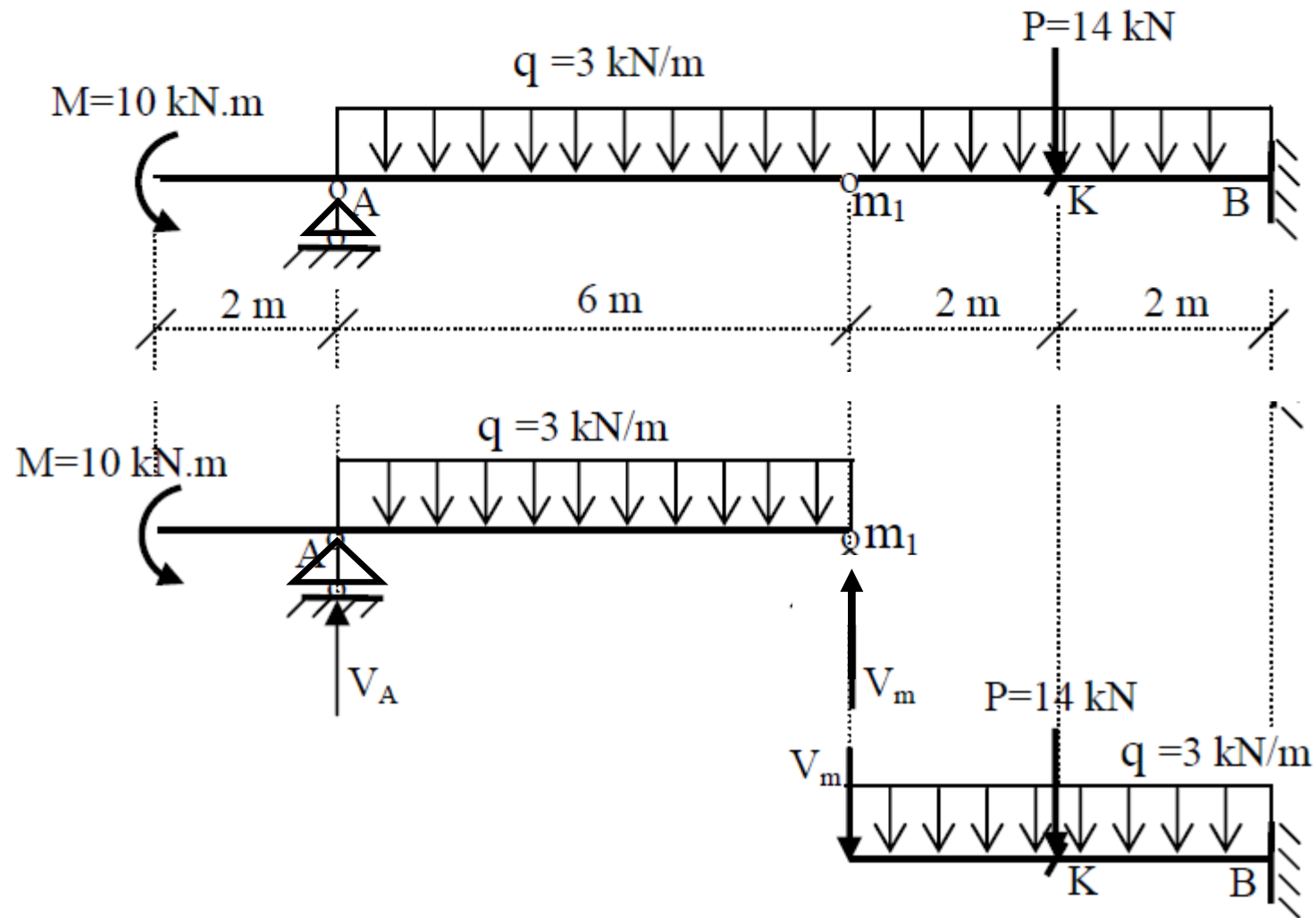


$$\sum Y = V_A + V_B + V_C - P_1 - q \cdot 8 - P - q \cdot 1 = 22 + 25,7143 + 8,2857 - 8 - 4 \cdot 8 - 12 - 4 \cdot 1 = 0$$

$$\sum M_{m_1} = -P_1 \cdot (7,5) + V_A \cdot 6 - q \cdot 6 \cdot 3 + q \cdot 2 \cdot 1 - V_B \cdot 2 + P \cdot 6 + M - V_C \cdot 9 + q \cdot 1 \cdot (9,5) = 0 \Rightarrow$$

$$-8 \cdot (7,5) + 22 \cdot 6 - 4 \cdot 6 \cdot 3 + 4 \cdot 2 \cdot 1 - (25,7143) \cdot 2 + 12 \cdot 6 + 8 - (8,2857) \cdot 9 + 4 \cdot 1 \cdot (9,5) = 0$$

ÖRNEK 3



Am₁- kirişinin

Mesnet tepkilerinin bulunması:

$$\sum M_{m1} = -M + V_A \cdot 6 - q \cdot 6 \cdot 3 = 0 \Rightarrow$$

$$V_A = \frac{10 + 3 \cdot 6 \cdot 3}{6} = 10,6667 \text{ kN}.$$

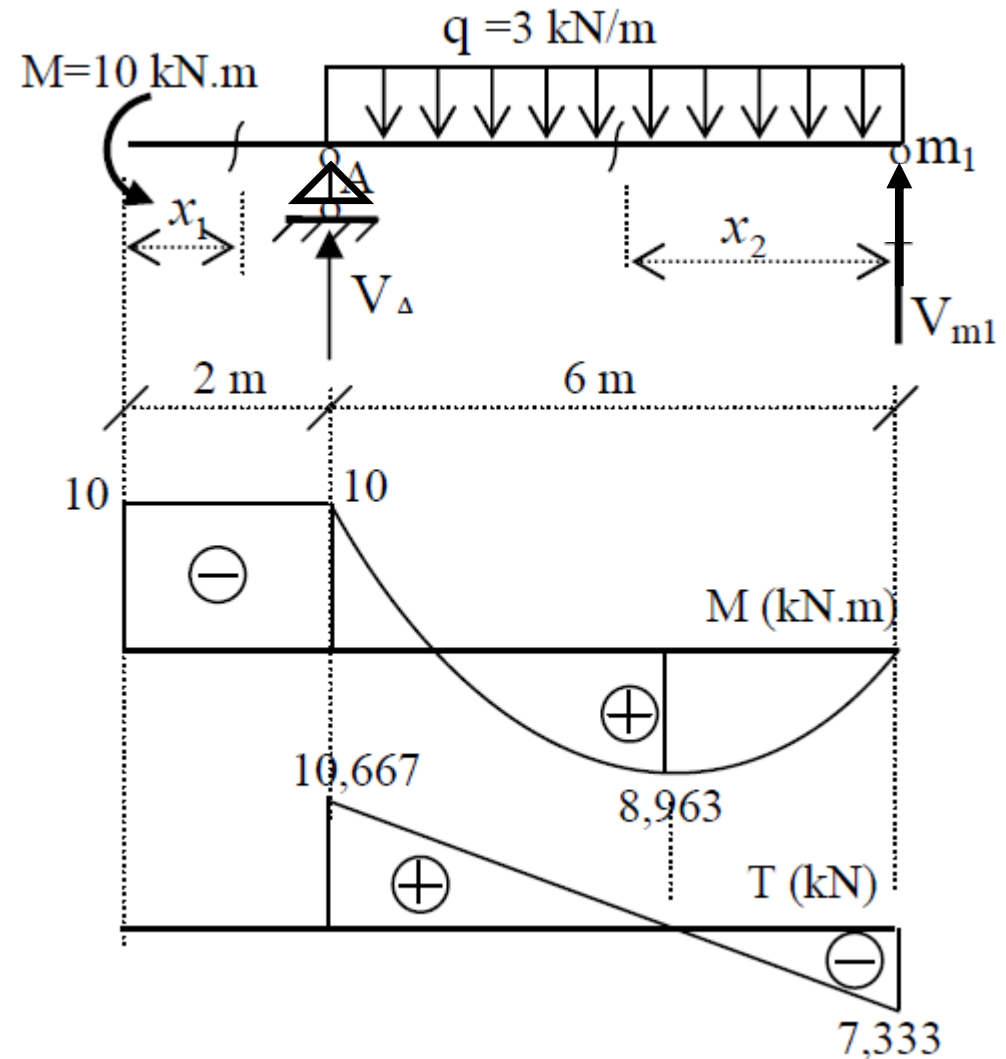
$$\sum M_A = -V_{m1} \cdot 6 + q \cdot 6 \cdot 3 - M = 0 \Rightarrow$$

$$V_{m1} = \frac{3 \cdot 6 \cdot 3 - 10}{6} = 7,3333 \text{ kN}.$$

Kontrol

$$\sum Y = V_A + V_{m1} - q \cdot 6 = 0 \Rightarrow$$

$$10,6667 + 7,3333 - 3 \cdot 6 = 0$$



$$0 \leq x_1 \leq 2$$

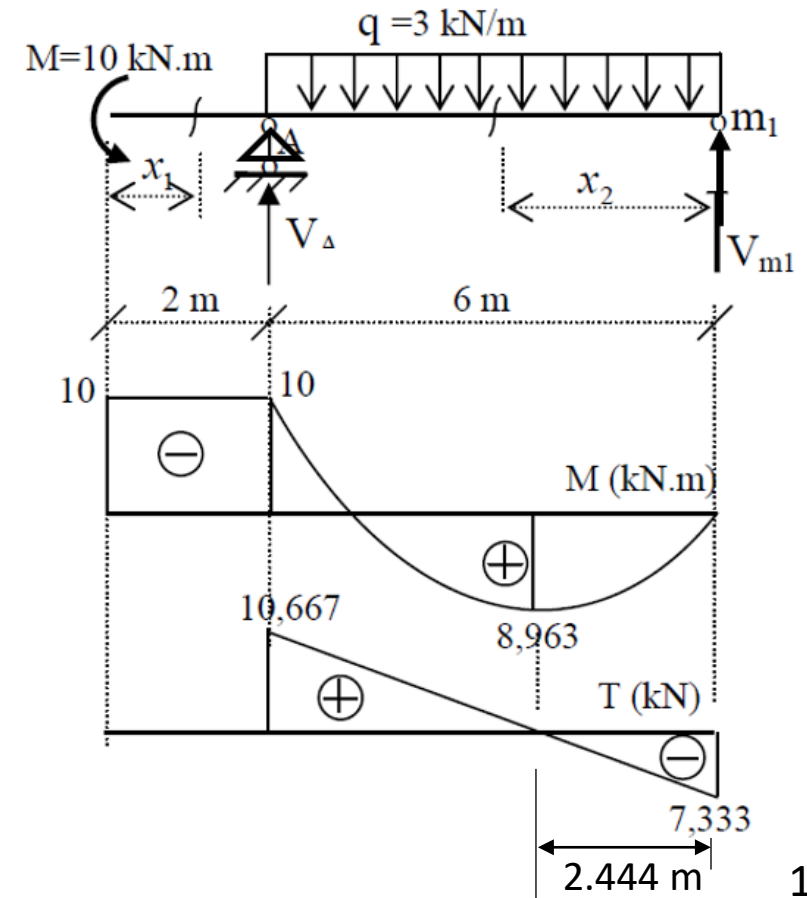
$$M = -10 \text{ kN.m}; \quad T = 0.$$

$$0 \leq x_2 \leq 6 \quad V_{m1} = 7.3333 \text{ kN}$$

$$x_2 = 6 \text{ m} \Rightarrow M_{x2} = (7.3333).6 - 3 \cdot \frac{6^2}{2} = -10 \text{ kN}, \quad T_{x2} = -7.3333 + 3.6 = 10.6667 \text{ kN}.$$

$$T_{x2} = -V_{m1} + qx_2 = 0 \Rightarrow x_2 = \frac{7.3333}{3} = 2.4444 \text{ m}.$$

$$M_{\max} = 7.3333.(2.4444) - 3 \cdot \frac{(2.444)^2}{2} = 8.963 \text{ kN.m}$$



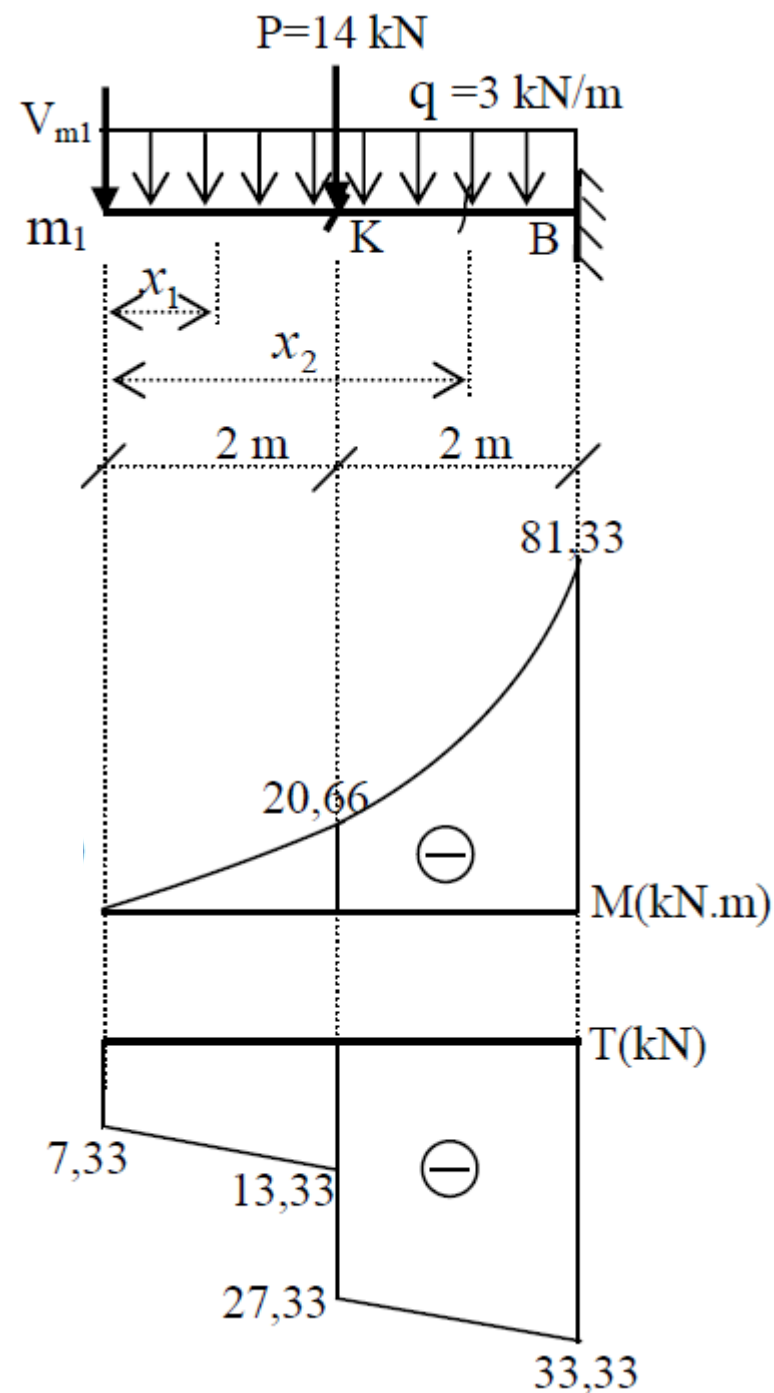
$$0 \leq x_1 \leq 2$$

$$M_{x1} = -V_{m1} \cdot x_1 - q \cdot \frac{x_1^2}{2}; \quad T_{x1} = -V_{m1} - q \cdot x_1.$$

$$x_1 = 0 \Rightarrow M_{x1} = 0, \quad T_{x1} = -V_{m1} = -7,333 \text{ kN}.$$

$$x_1 = 2 \text{ m} \Rightarrow M_{x1} = -(7,333) \cdot 2 - 3 \cdot \frac{2^2}{2} = -20,666 \text{ kN.m}$$

$$T_{x1} = -7,3333 - 3 \cdot 2 = -13,3333 \text{ kN}.$$



$$2 \leq x_2 \leq 4$$

$$M_{x_2} = -V_{m1} \cdot x_2 - q \cdot \frac{x_2^2}{2} - P \cdot (x_2 - 2);$$

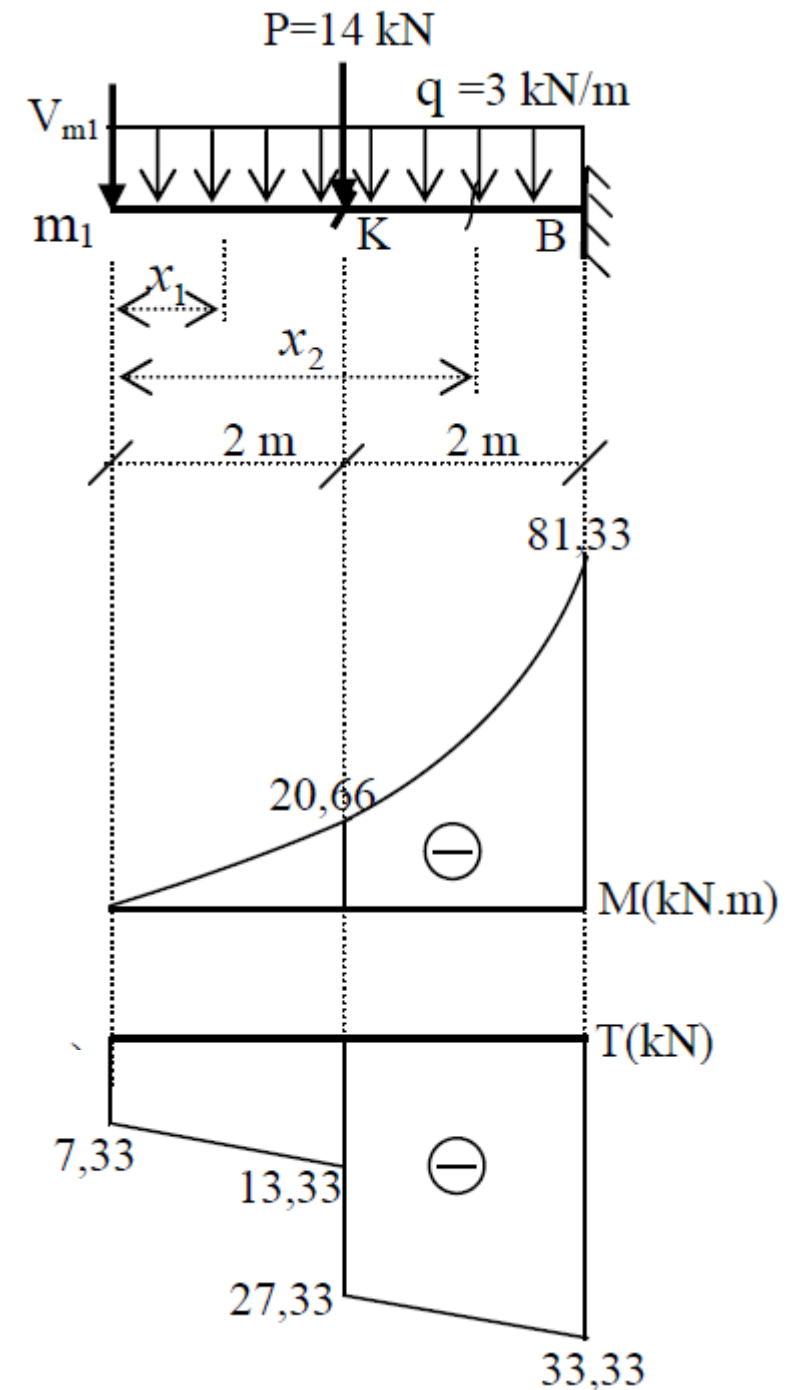
$$T_{x_2} = -V_{m1} - q \cdot x_2 - P.$$

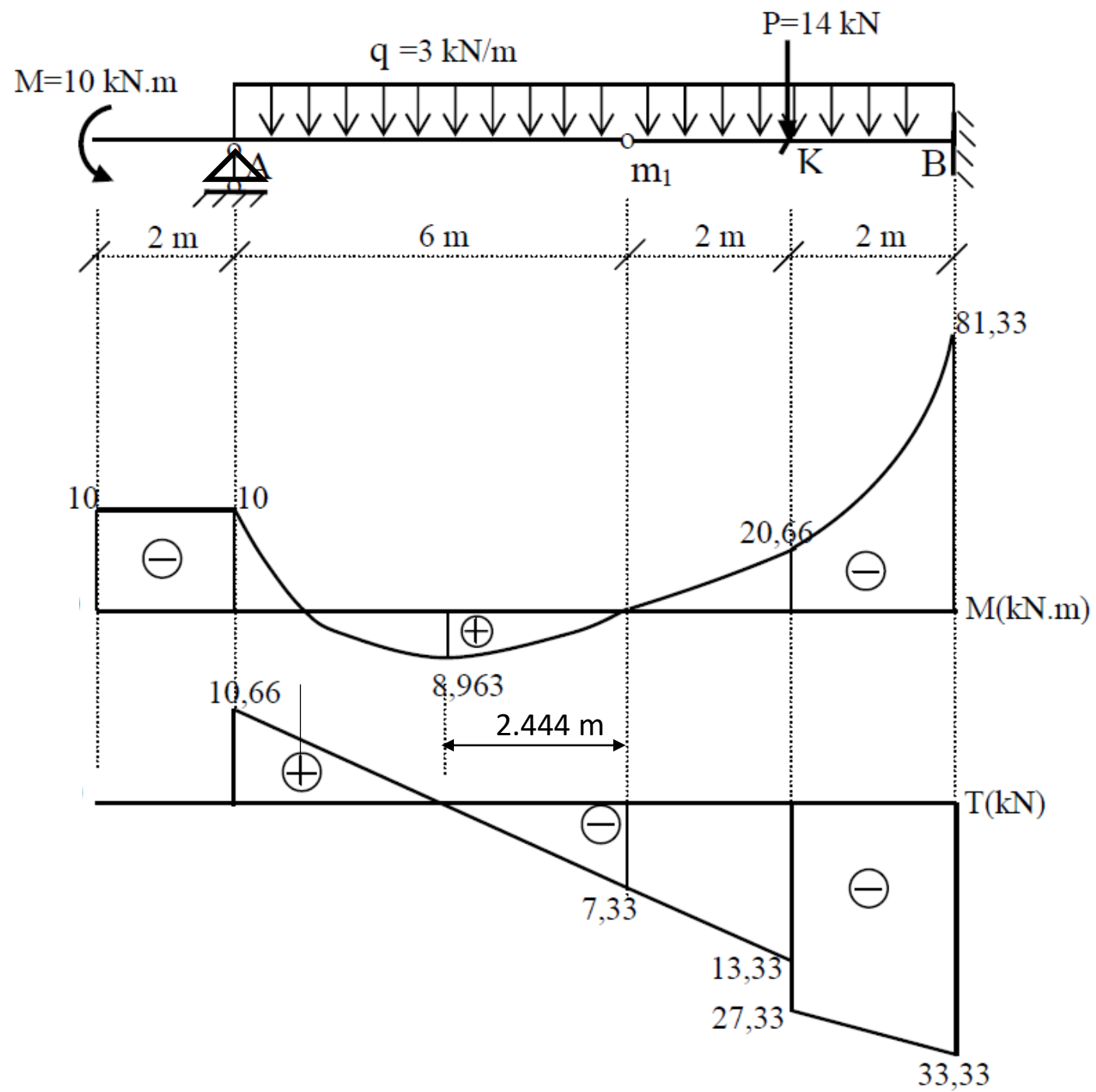
$$x_2 = 2 \text{ m} \Rightarrow M_{x_2} = -20,6666 \text{ kN.m}$$

$$T_{x_2} = -7,3333 - 3 \cdot 2 - 14 = -27,3333 \text{ kN}.$$

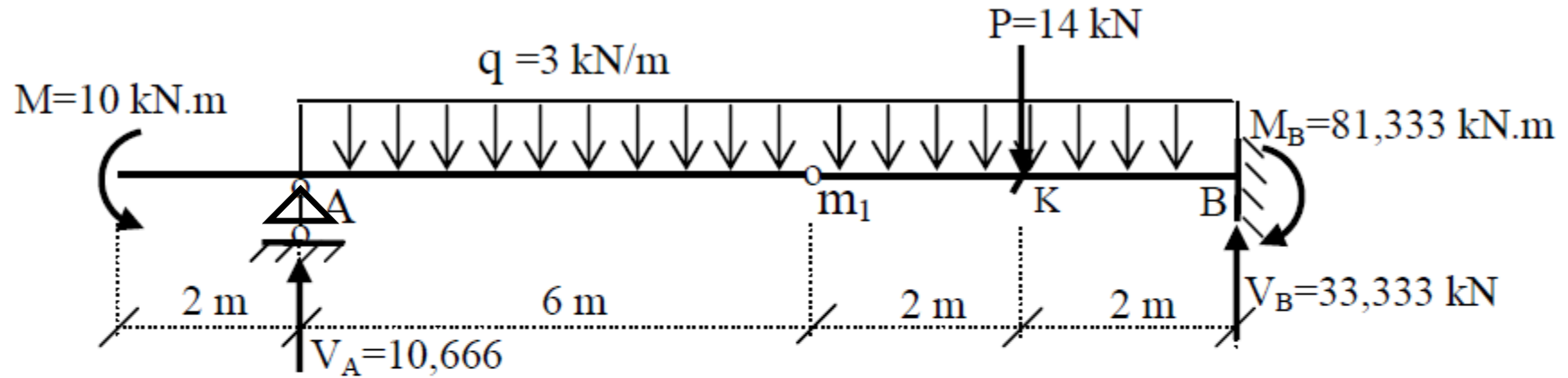
$$\begin{aligned} x_2 = 4 \text{ m} \Rightarrow M_{x_2} &= -(7,3333) \cdot 4 - 3 \cdot \frac{4^2}{2} - 14 \cdot 2 = \\ &= -81,333 \text{ kN.m} = M_B \end{aligned}$$

$$x_2 = 4 \text{ m} \rightarrow T_{x_2} = -7.3333 - 3 \cdot 4 - 14 = -33,33 \text{ kN}$$





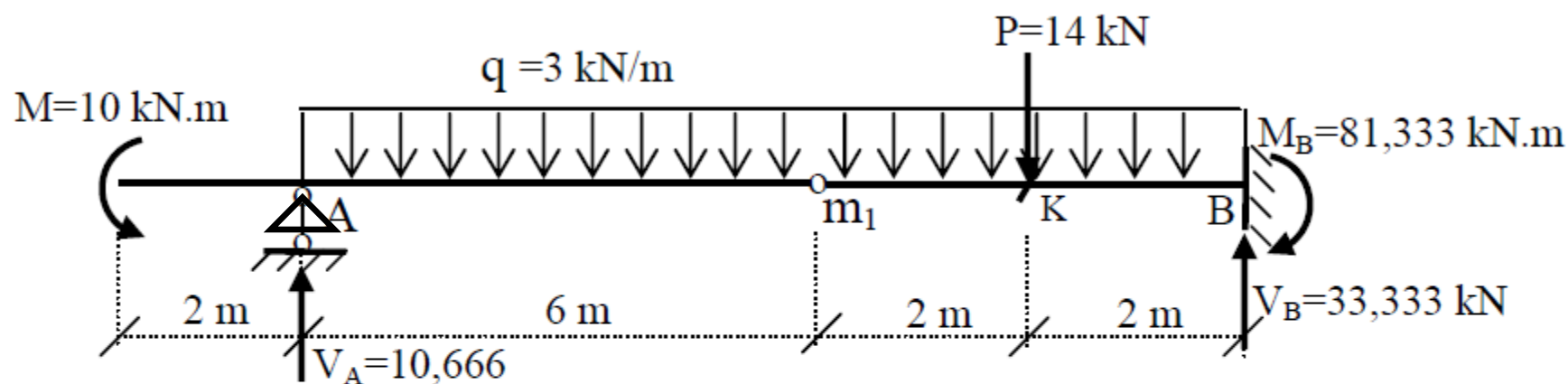
Kirişin tümünün denge kontrolü



$$\sum Y = V_A - q \cdot 10 - P - V_B = 0 \Rightarrow 10,6667 - 3 \cdot 10 - 14 + 33,3333 = 0$$

$$\begin{aligned} \sum M_{m1} = -M + V_A \cdot 6 - q \cdot 6 \cdot 3 + q \cdot 4 \cdot 2 + P \cdot 2 - V_B \cdot 4 + M_B = 0 \Rightarrow \\ -10 + 10,6667 \cdot 6 + 3 \cdot 4 \cdot 2 + 14 \cdot 2 - (33,3333) \cdot 4 + 81,3333 = 0 \end{aligned}$$

Kirişin tümünün denge kontrolü

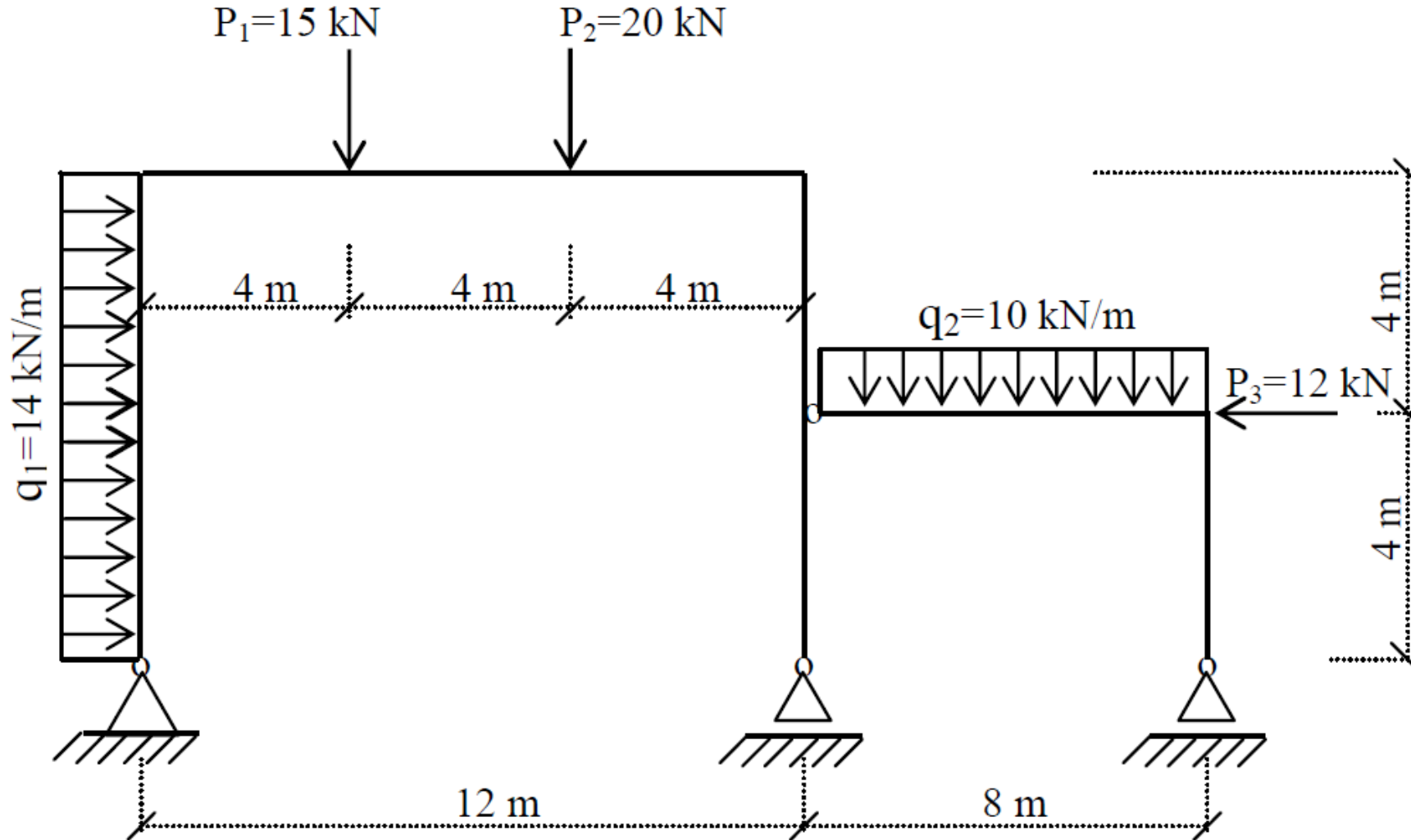


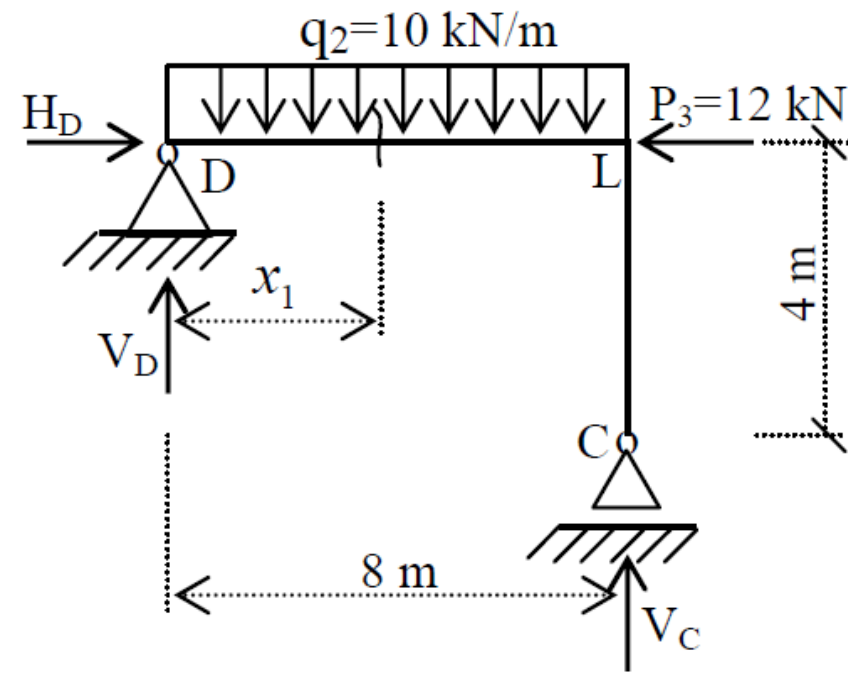
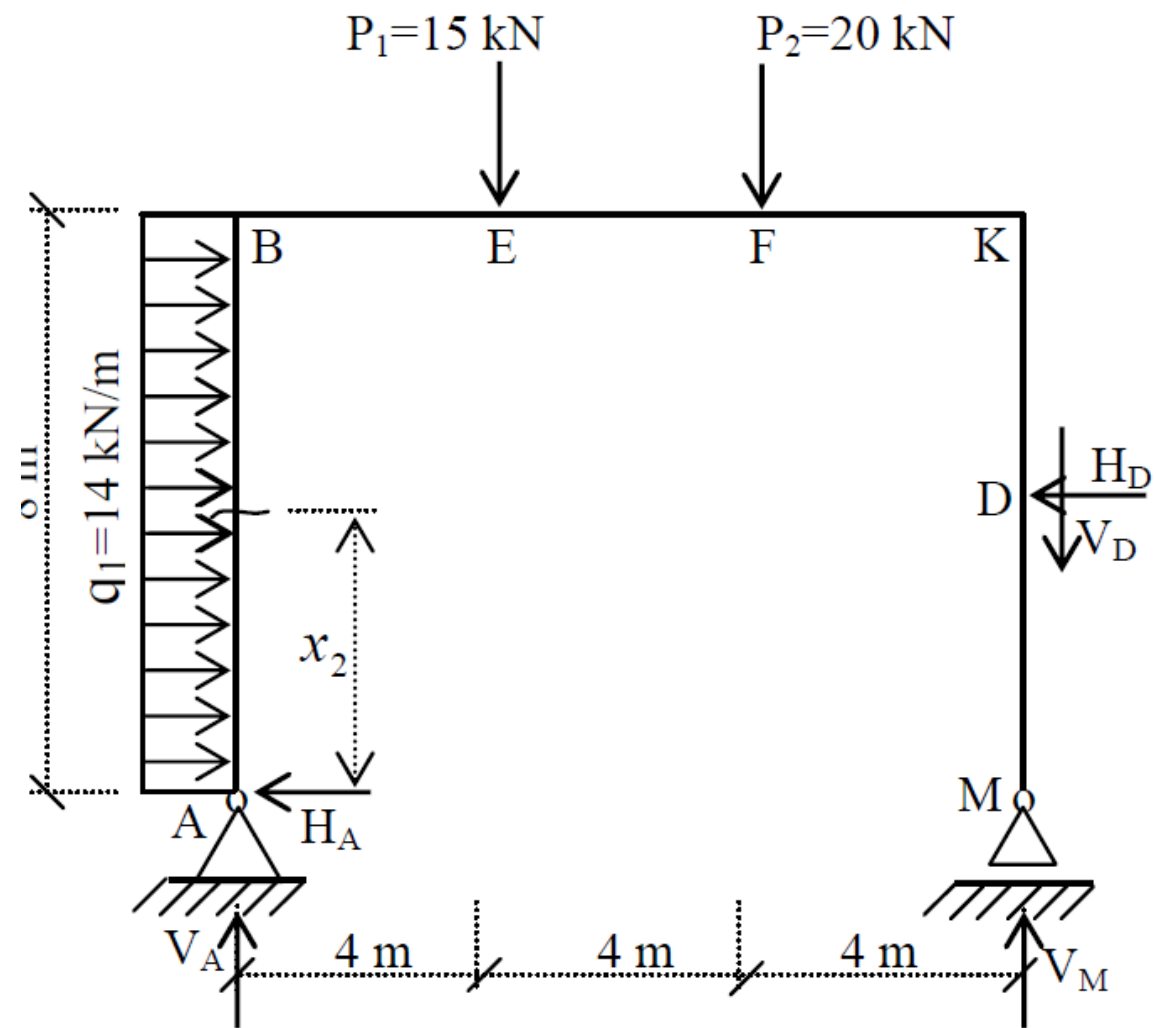
$$\sum Y = V_A - q \cdot 10 - P - V_B = 0 \Rightarrow 10,6667 - 3 \cdot 10 - 14 + 33,3333 = 0$$

$$\begin{aligned} \sum M_{m1} = -M + V_A \cdot 6 - q \cdot 6 \cdot 3 + q \cdot 4 \cdot 2 + P \cdot 2 - V_B \cdot 4 + M_B = 0 \Rightarrow \\ -10 + 10,6667 \cdot 6 + 3 \cdot 4 \cdot 2 + 14 \cdot 2 - (33,3333) \cdot 4 + 81,3333 = 0 \end{aligned}$$

ÖRNEK 4

Şekilde verilen çerçevenin M, N, T diyagramlarını çiziniz





$$\sum X = H_D - P_3 = 0 \Rightarrow H_D = P_3 = 12 \text{ kN} ; \quad V_D = V_C = \frac{q_2 \cdot 8}{2} = \frac{10 \cdot 8}{2} = 40 \text{ kN}.$$

Bölgeler için iç kuvvetlerin (M , T ve N) değerlerinin hesabı:.

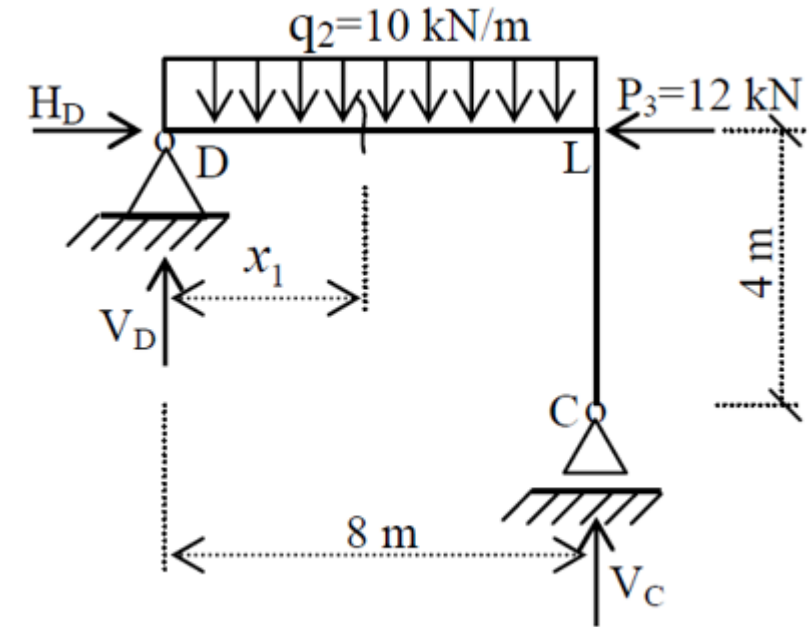
DL bölgesi $0 \leq x_1 \leq 8$

$$M_{x1} = V_D \cdot x_1 - q_2 \cdot \frac{x_1^2}{2} ; \quad T_{x1} = V_D - q_2 \cdot x_1 ; \quad N = -H_D = -12 \text{ kN}.$$

$$x_1 = 0 \Rightarrow M_{x1} = 0 ; \quad T_{x1} = V_D = 40 \text{ kN}$$

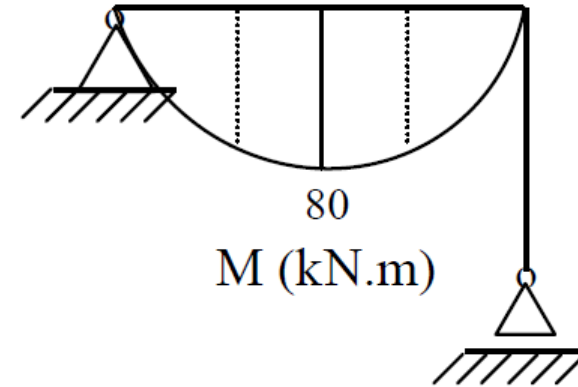
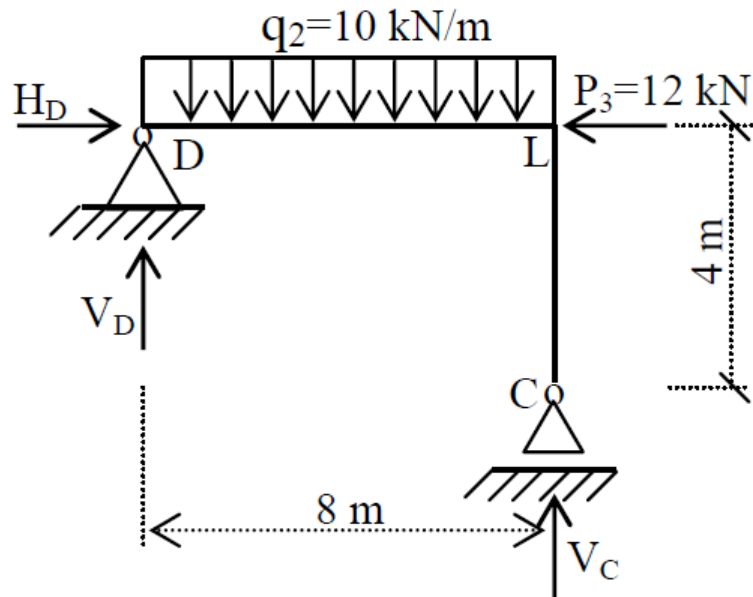
$$x_1 = 4 \text{ m} \Rightarrow M_{x1}^{\max} = \frac{q_2 \cdot 8^2}{2} = \frac{10 \cdot 64}{2} = 80 \text{ kN.m} ; \quad T_{x1} = 0.$$

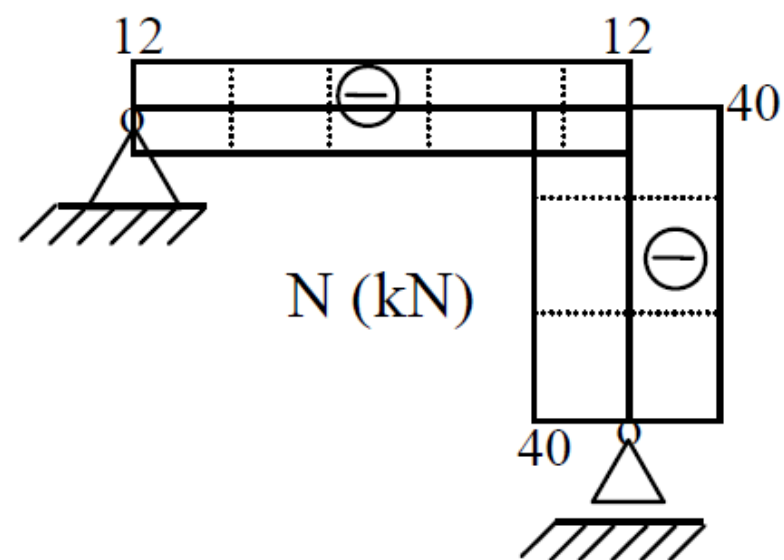
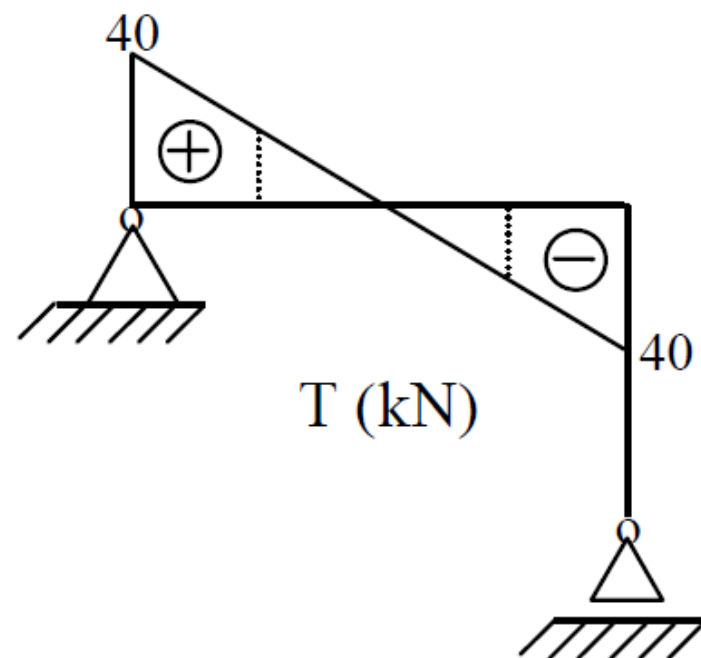
$$x_1 = 8 \text{ m} \Rightarrow M_{x1} = 40 \cdot 8 - 10 \cdot \frac{8^2}{2} = 0 ; \quad T_{x1} = 40 - 10 \cdot 8 = -40 \text{ kN}.$$



CL bölgesinde

$$M = 0; \quad T = 0; \quad N = -V_C = -40 \text{ kN}.$$





Mesnet tepkileri

$$\sum X = q_1 \cdot 8 - H_A - H_D = 0 \Rightarrow H_A = 14.8 - 12 = 100 \text{ kN}$$

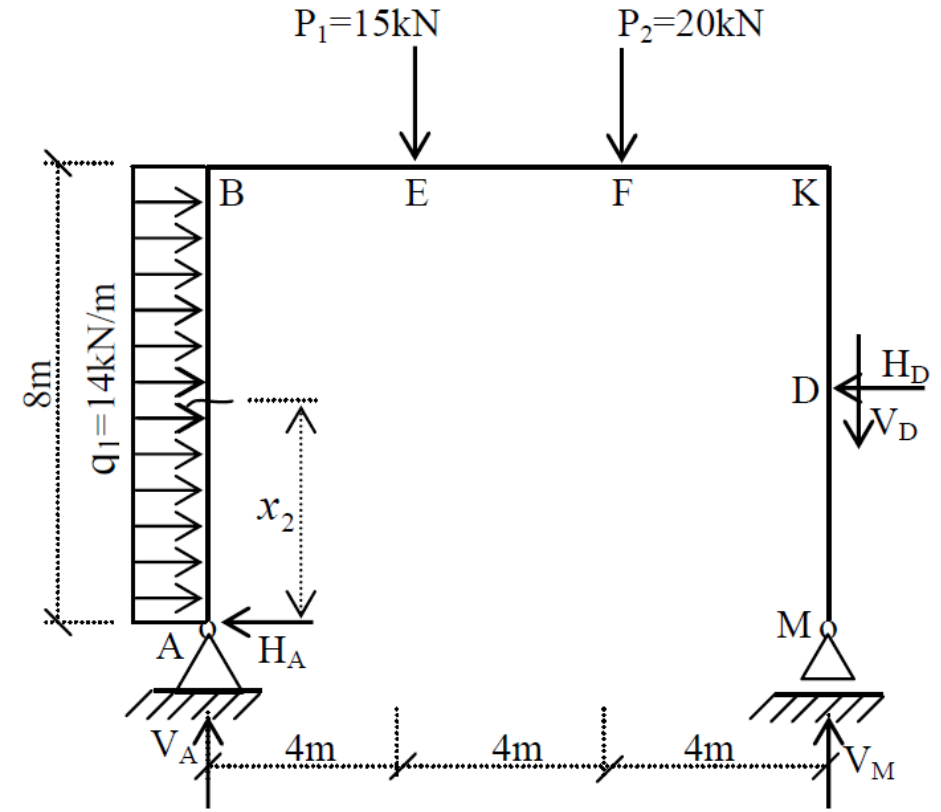
$$\sum M_D = V_A \cdot 12 + q_1 \cdot 8 \cdot 4 - P_1 \cdot 8 - P_2 \cdot 4 - H_D \cdot 4 = 0 \Rightarrow$$

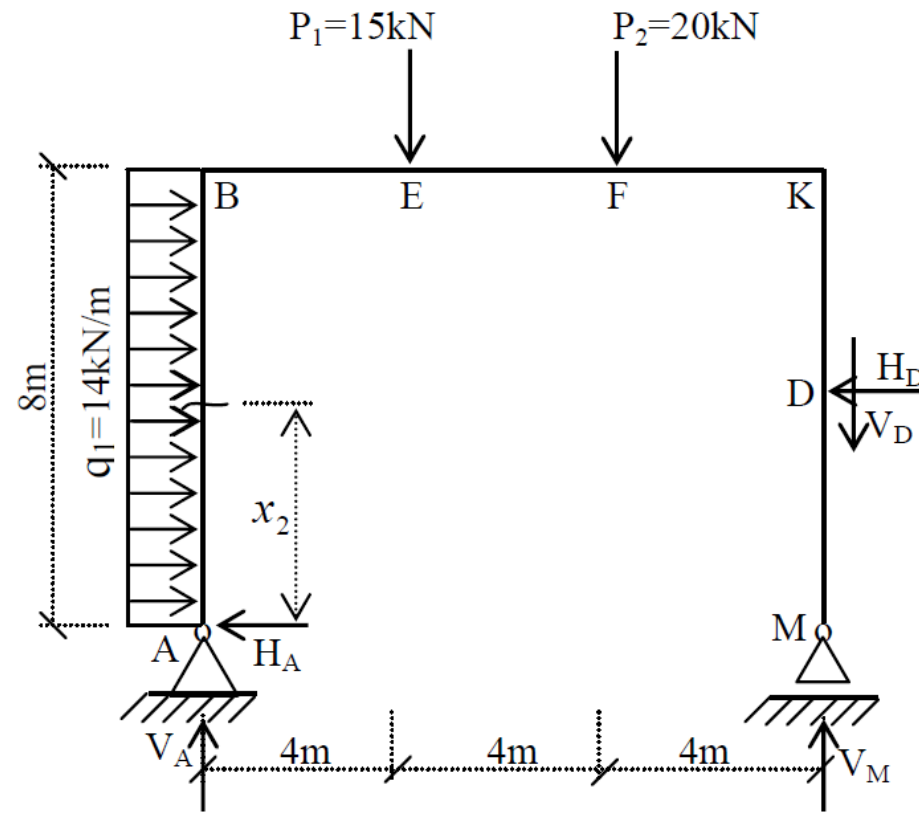
$$V_A = \frac{-14.8 \cdot 4 + 15 \cdot 8 + 20 \cdot 4 + 12 \cdot 4}{12} = -16,6667 \text{ kN}$$

$$\sum M_A = -V_M \cdot 12 - H_D \cdot 4 + P_1 \cdot 4 + P_2 \cdot 8 + q_1 \cdot 8 \cdot 4 + V_D \cdot 12 = 0 \Rightarrow$$

$$V_M = \frac{-12 \cdot 4 + 15 \cdot 4 + 20 \cdot 8 + 14.8 \cdot 4 + 40 \cdot 12}{12} = 91,6667 \text{ kN}.$$

$$\sum Y = V_A - P_1 - P_2 - V_D + V_M = -16,6667 - 15 - 20 - 40 + 91,6667 = 0$$





AB bölgesi $0 \leq x_2 \leq 8$

$$M_{x_2} = H_A \cdot x_2 - q_1 \cdot \frac{x_2^2}{2}; \quad T_{x_1} = H_A - q_1 \cdot x_2; \quad N = -V_A = -(-16,666) = 16,6667 \text{ kN}$$

$$x_2 = 0 \Rightarrow M_{x_2} = 0; \quad T_{x_2} = H_A = 100 \text{ kN}.$$

$$x_2 = 4 \text{ m} \Rightarrow M_{x_2} = 100 \cdot 4 - 14 \cdot \frac{4^2}{2} = 288 \text{ kN.m}$$

$$x_2 = 8 \text{ m} \Rightarrow M_{x_2} = 100 \cdot 8 - 14 \cdot \frac{8^2}{2} = 352 \text{ kN.m}; \quad T_{x_2} = 100 - 14 \cdot 8 = -12 \text{ kN}.$$

Eğilme momentinin maksimum değer aldığı kesitin yeri

$$T_{x_2} = 100 - 14 \cdot x_0 = 0 \Rightarrow x_0 = 7,142857 \text{ m}$$

$$M_{\max} = 100 \cdot (7,142857) - 14 \cdot \frac{(7,142857)^2}{2} = 357,142857 \text{ kN.m}$$

Çerçeve kirişinin B kesitinde

$$M_B = H_A \cdot 8 - q_1 \cdot \frac{8^2}{2} = 100 \cdot 8 - 14 \cdot \frac{64}{2} = 352 \text{ kN.m}$$

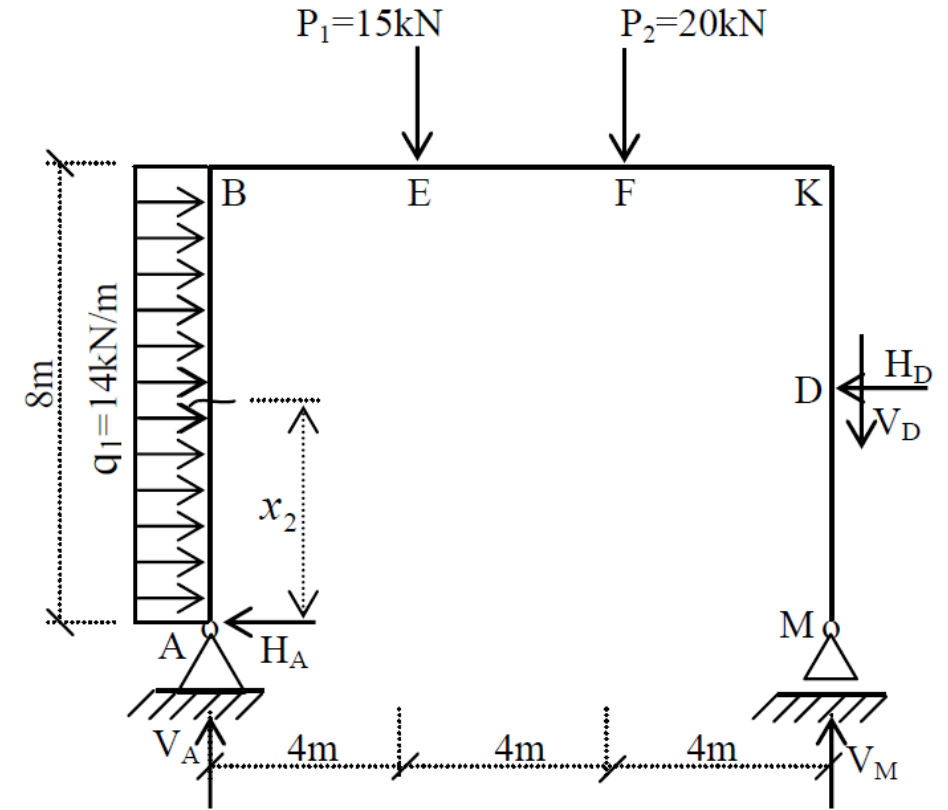
$$T_B = V_A = -16,6667 \text{ kN} = T_E^{sol}$$

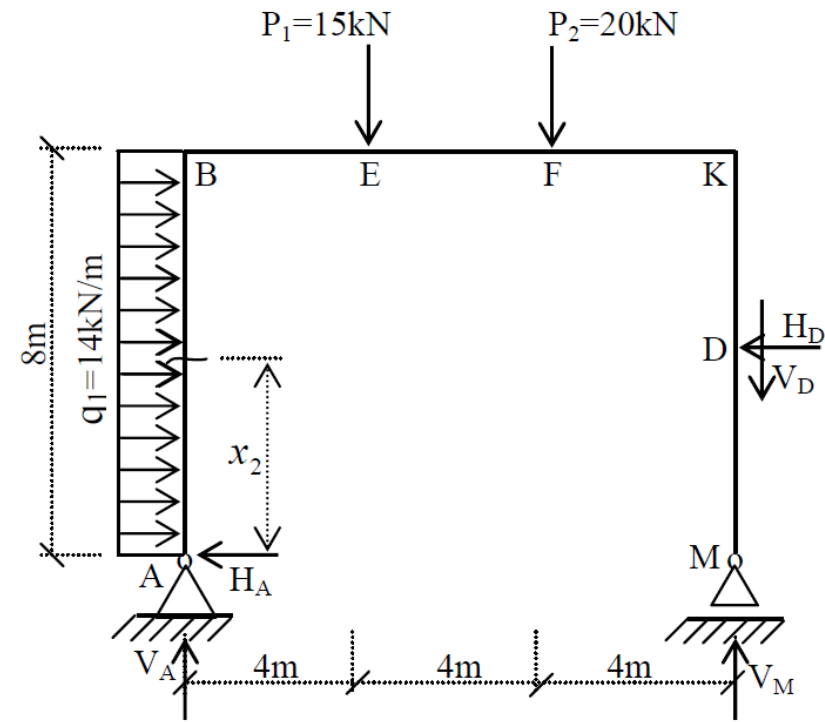
$$N = H_A - q_1 \cdot 8 = 100 - 14 \cdot 8 = -12 \text{ kN}$$

E kesitinde

$$M_E = H_A \cdot 8 - q_1 \cdot \frac{8^2}{2} + V_A \cdot 4 = 100 \cdot 8 - 14 \cdot \frac{64}{2} - (16,6667) \cdot 4 = 285,333 \text{ kN.m}$$

$$T_E^{sağ} = V_A - P_1 = -16,6667 - 15 = -31,6667 \text{ kN} = T_F^{sol}$$





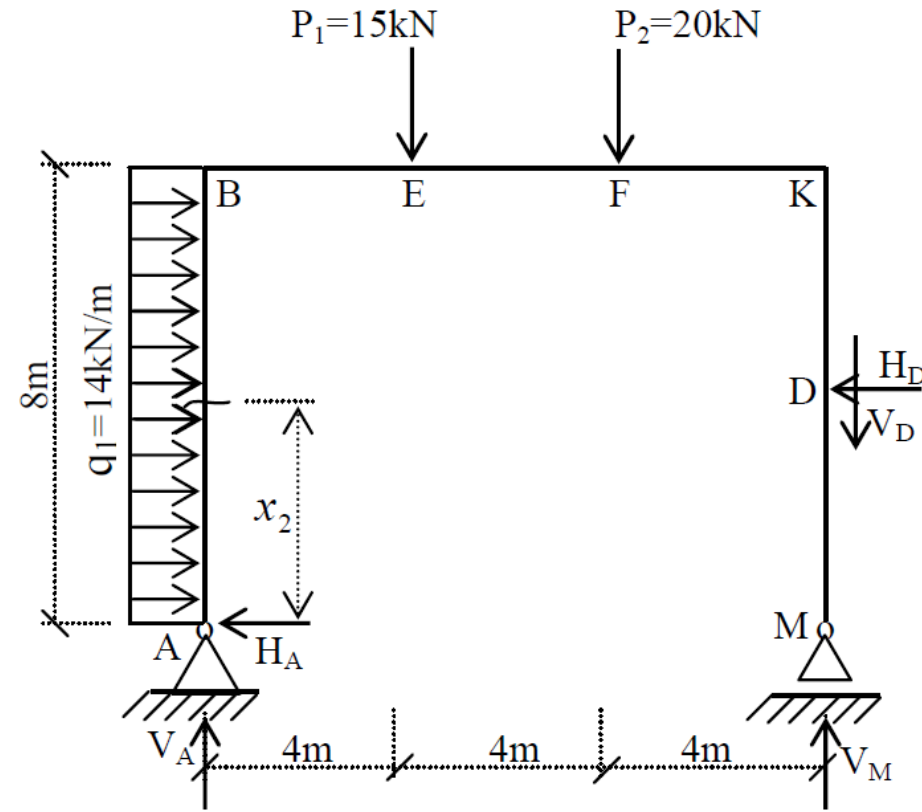
F kesitinde

$$M_F = H_A \cdot 8 - q_1 \cdot \frac{8^2}{2} + V_A \cdot 8 - P_1 \cdot 4 = 100 \cdot 8 - 14 \cdot \frac{64}{2} - (16,666) \cdot 8 - 15 \cdot 4 = 158,666 \text{ kN.m}$$

$$T_F^{sağ} = V_A - P_1 - P_2 = -16,6667 - 15 - 20 = -51,6667 \text{ kN} = T_K$$

K kesitinde

$$M_K = H_A \cdot 8 - q_1 \cdot \frac{8^2}{2} + V_A \cdot 12 - P_1 \cdot 8 - P_2 \cdot 4 = 100 \cdot 8 - 14 \cdot 32 - (16,6667) \cdot 12 - 15 \cdot 8 - 20 \cdot 4 = -48 \text{ kN.m}$$



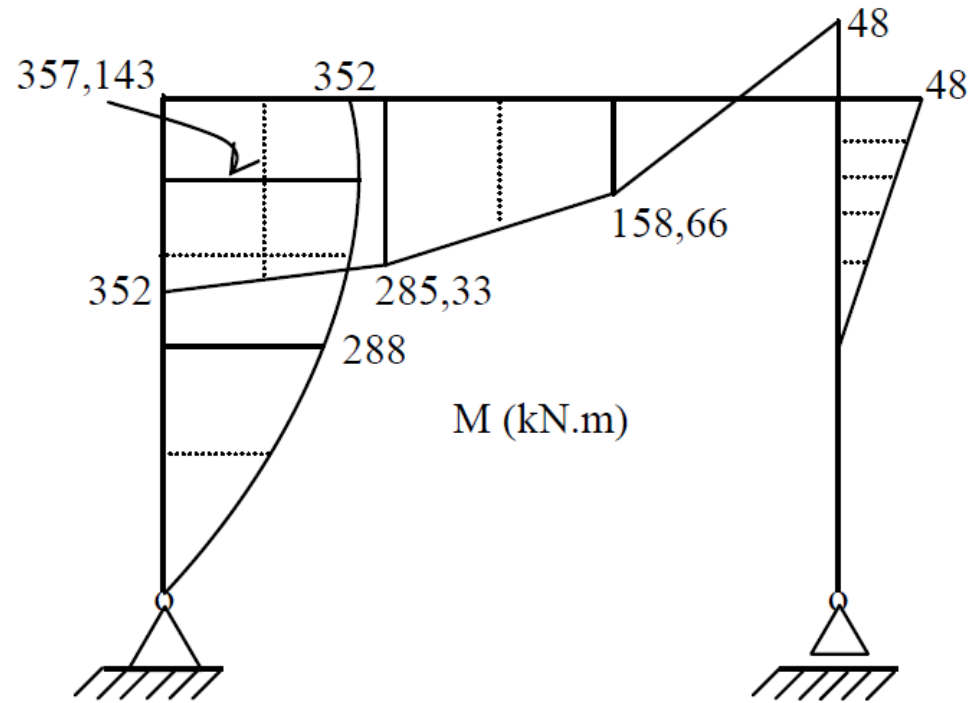
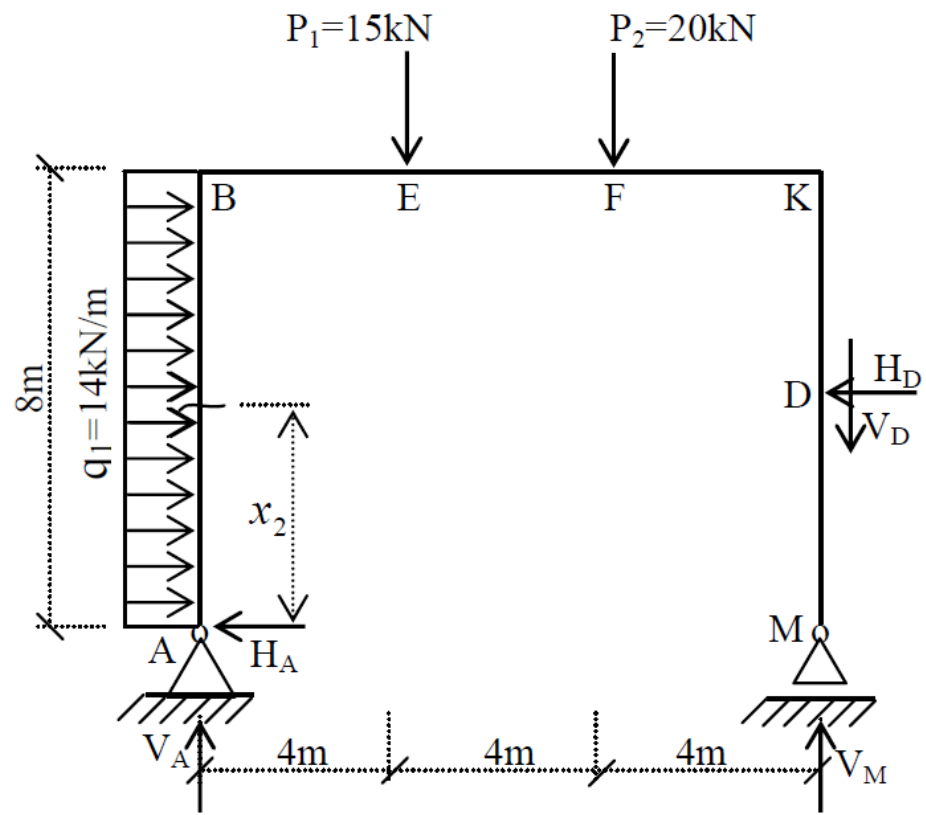
MK kolonu için :

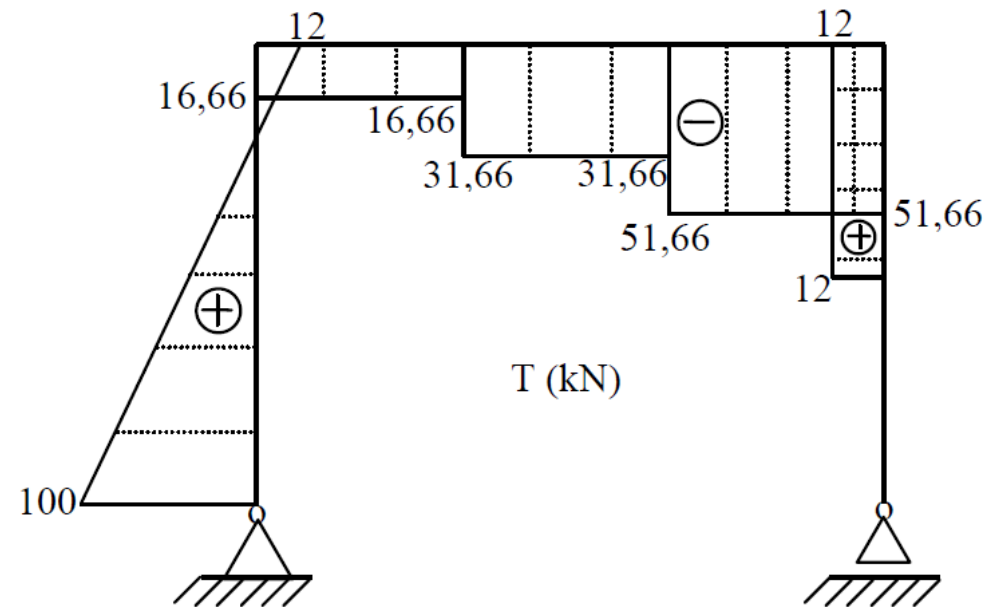
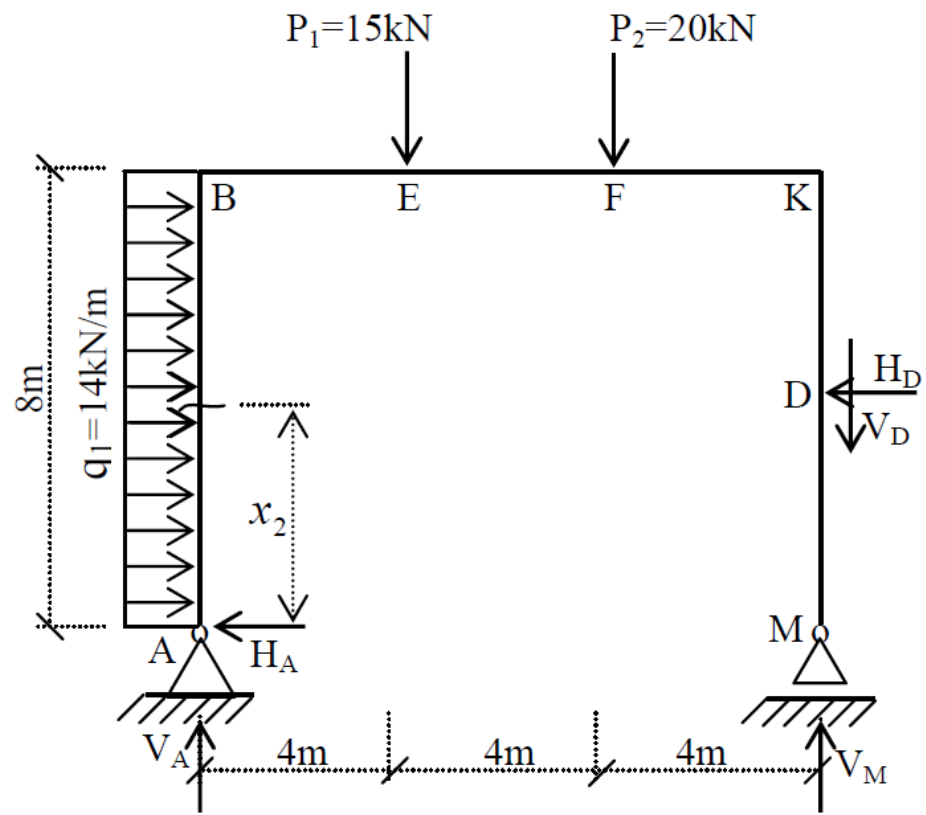
MD bölgesinde $M = 0$; $T = 0$; $N = -V_M = -91,6667 \text{ kN}$

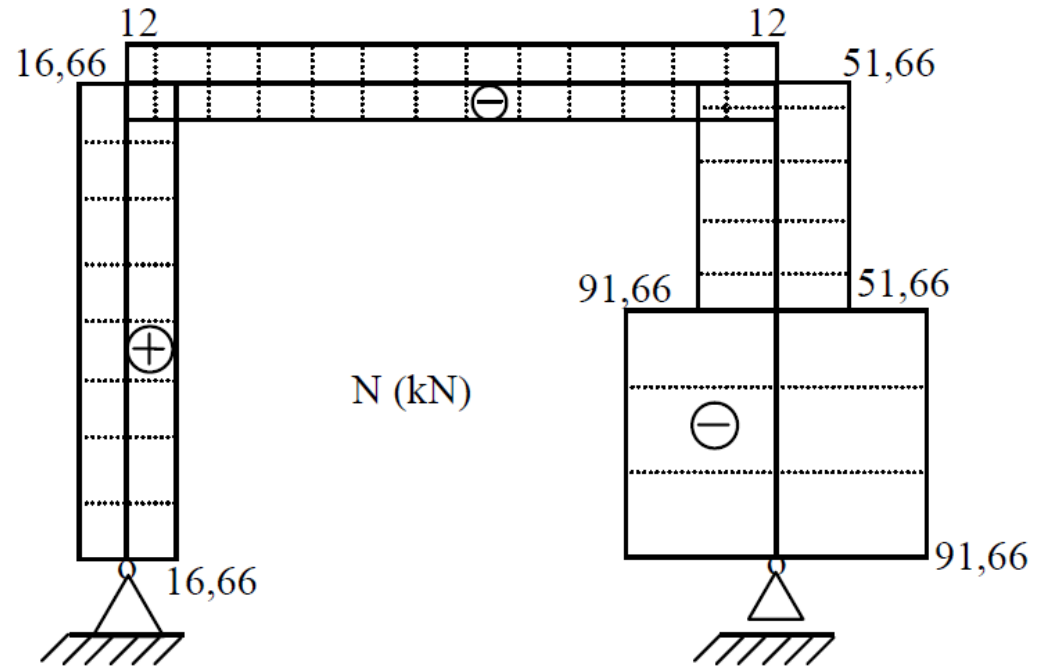
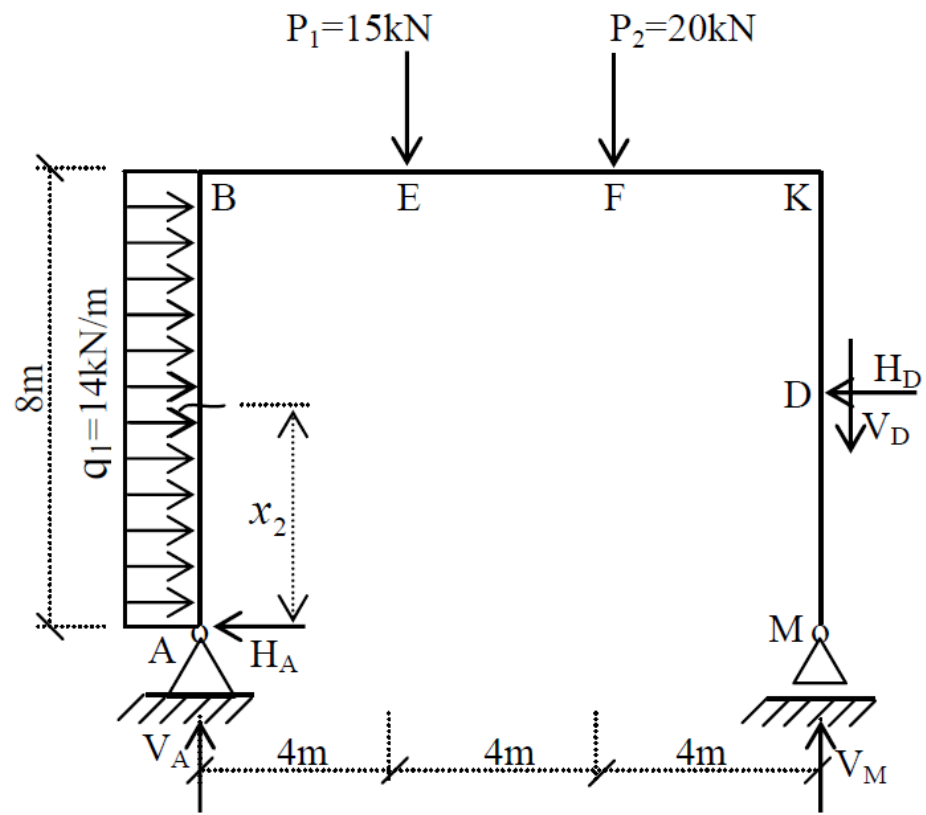
DK bölgesinde

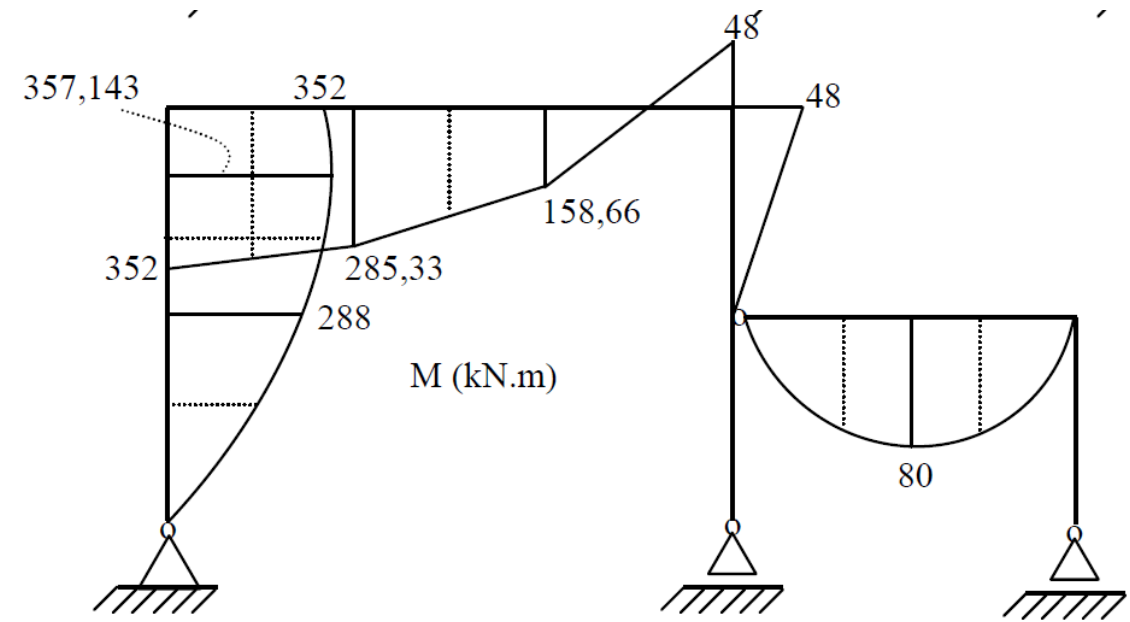
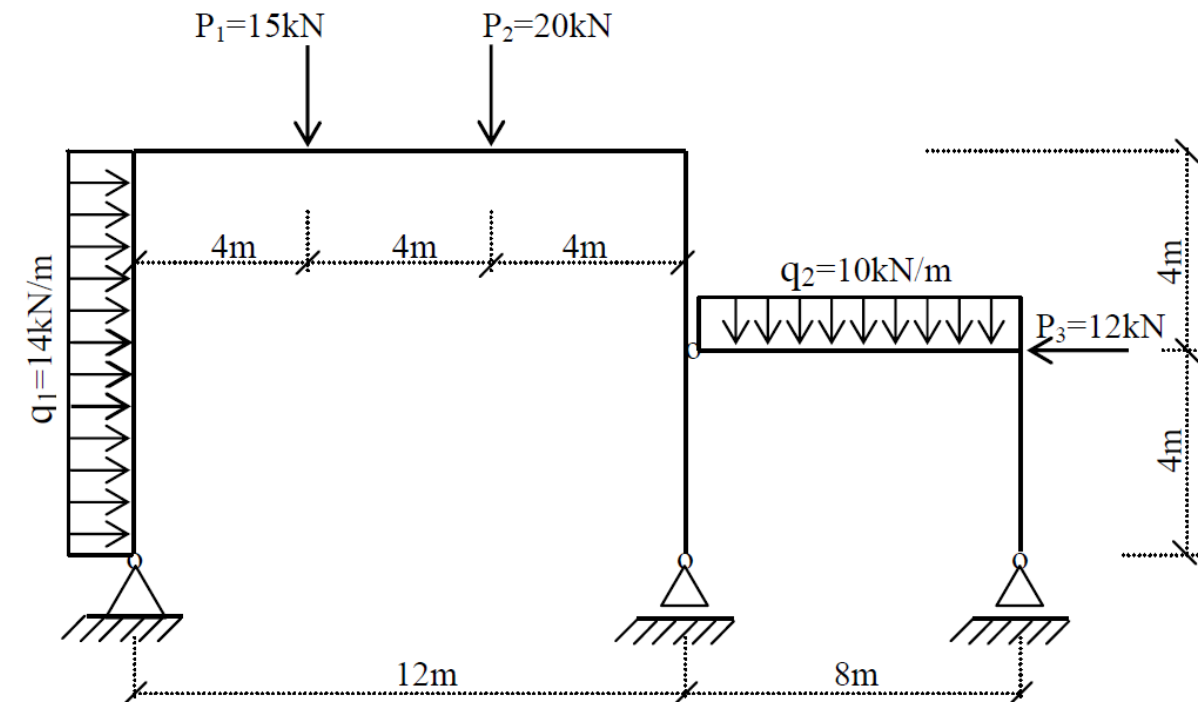
$$M_K = -H_B \cdot 4 = -12 \cdot 4 = -48 \text{ kN} ; \quad T_{DK} = H_D = 12 \text{ kN} ;$$

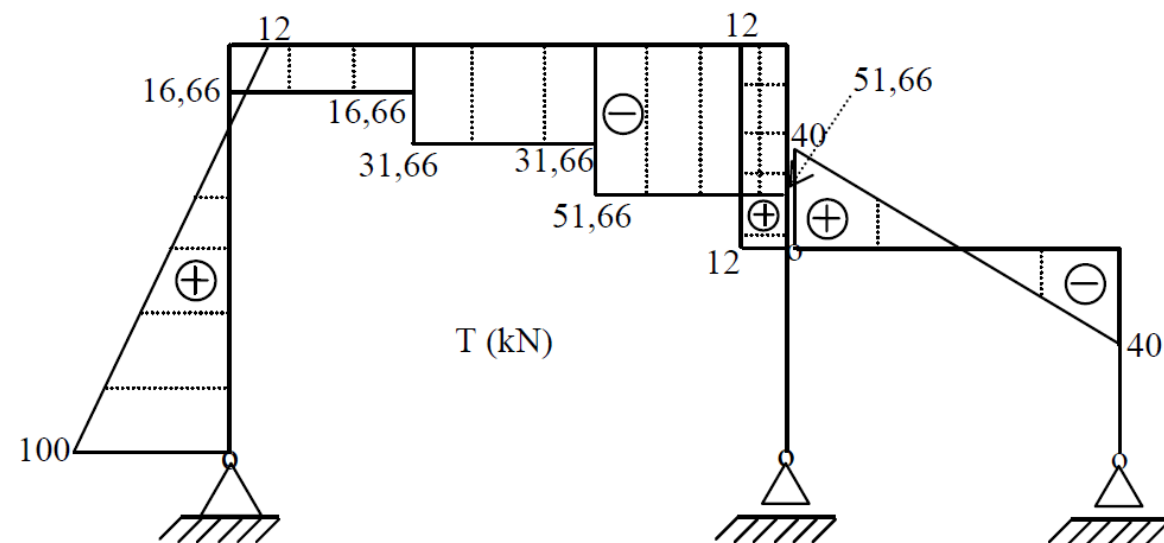
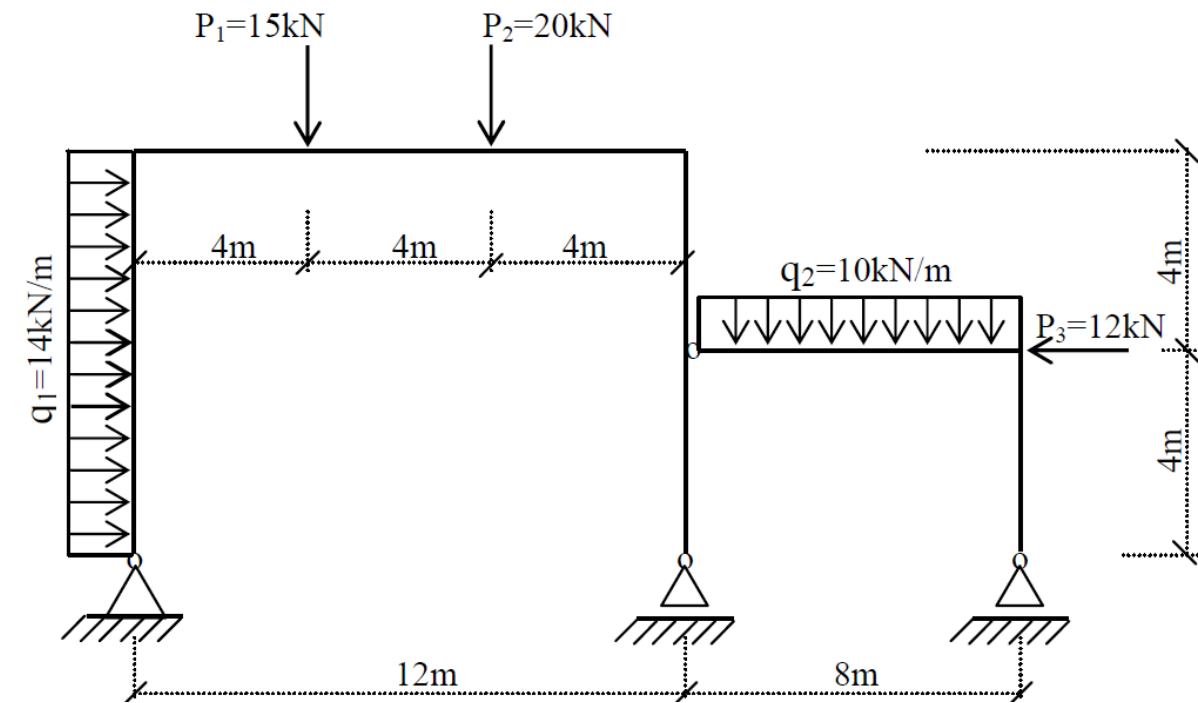
$$N_{DK} = -V_M + V_D = -91,6667 + 40 = -51,6667 \text{ kN}$$

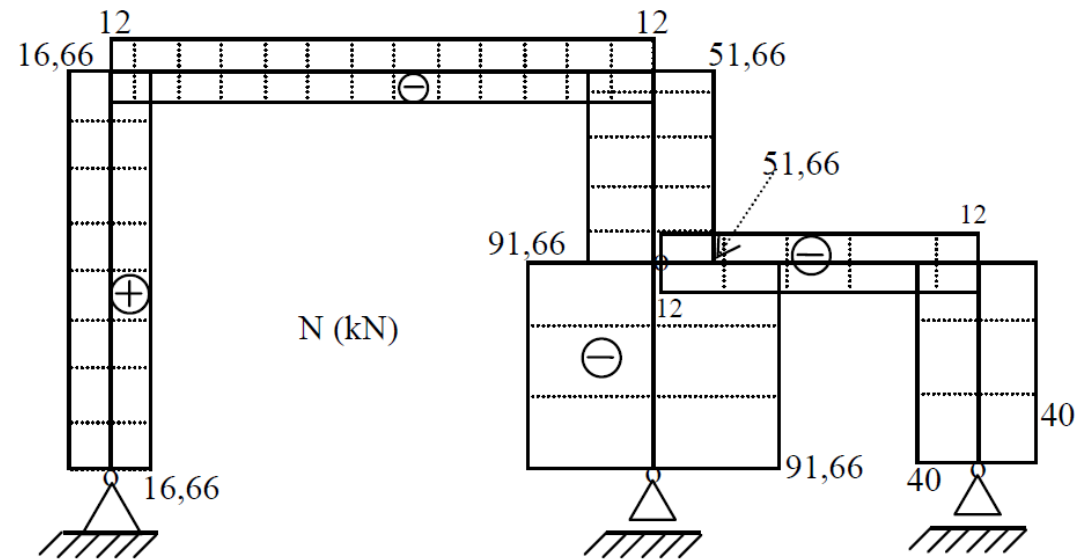
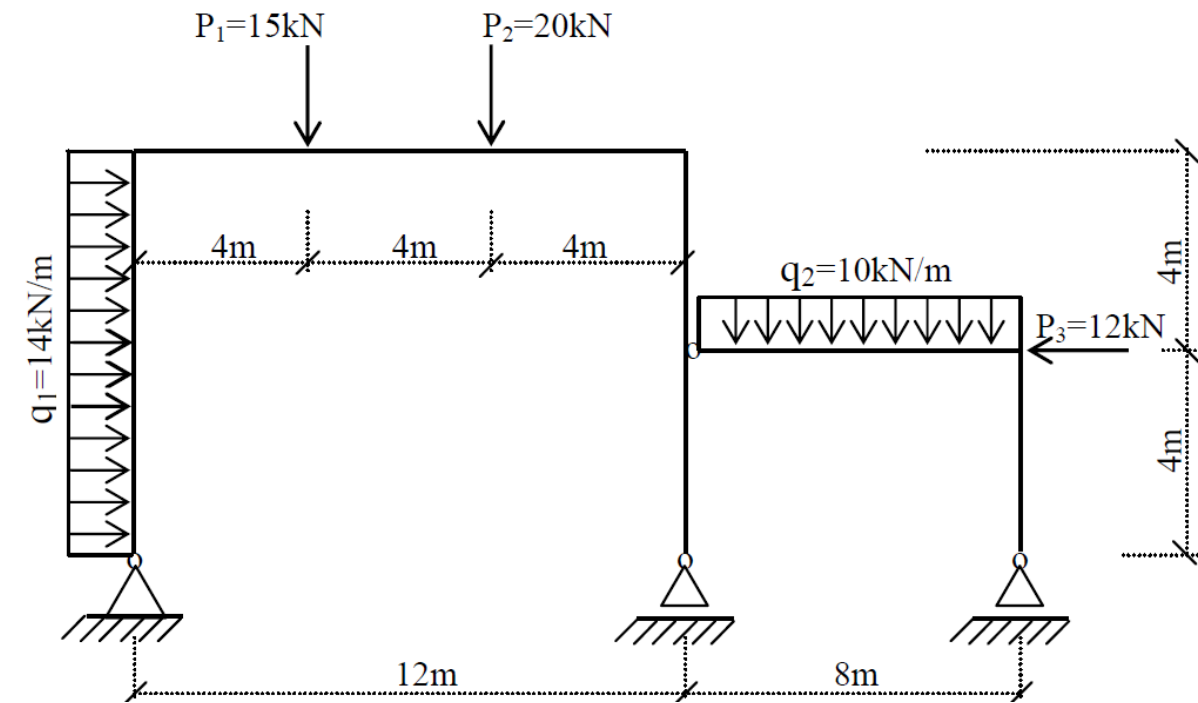








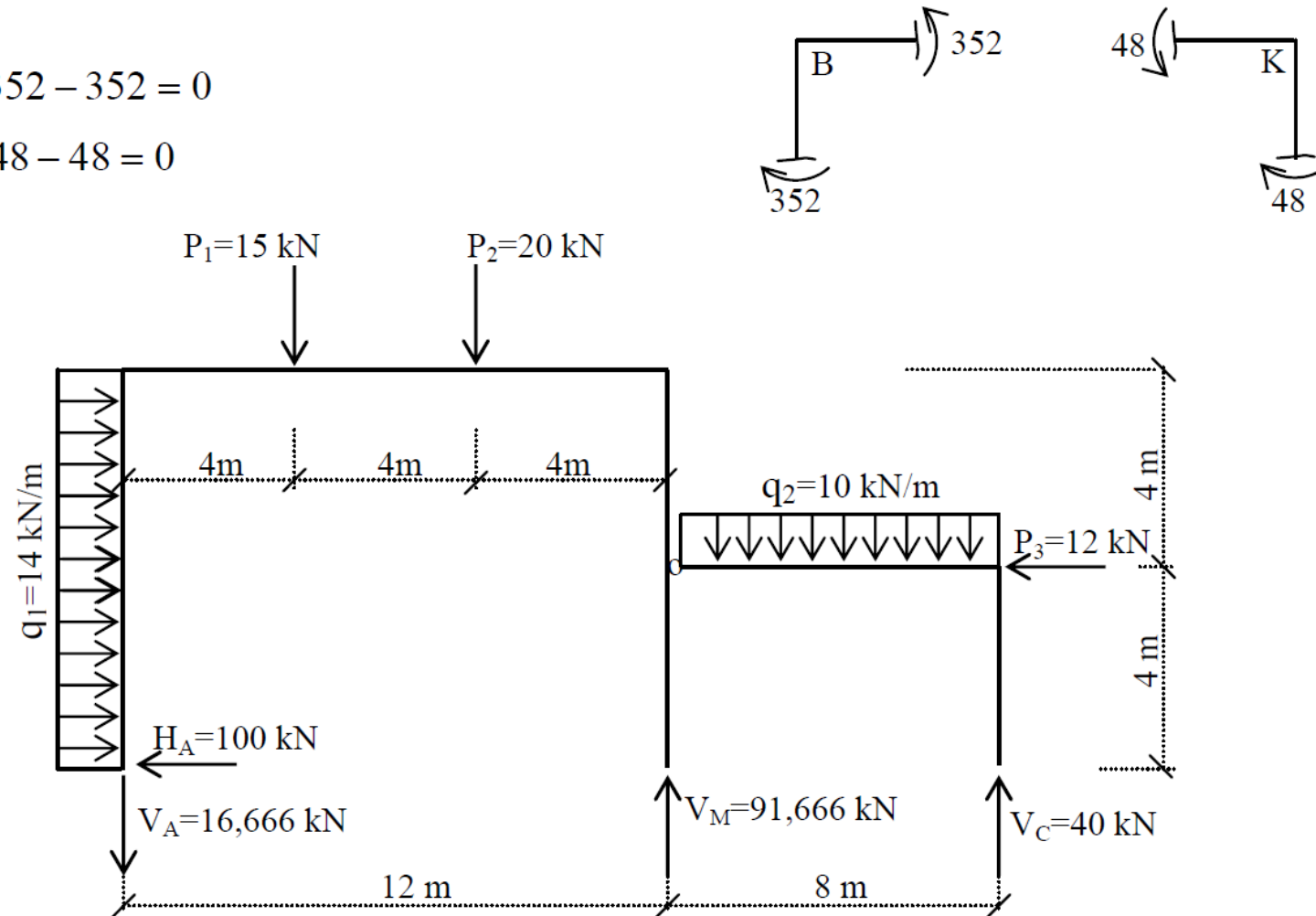




Kontrol :

$$\sum M_B = 352 - 352 = 0$$

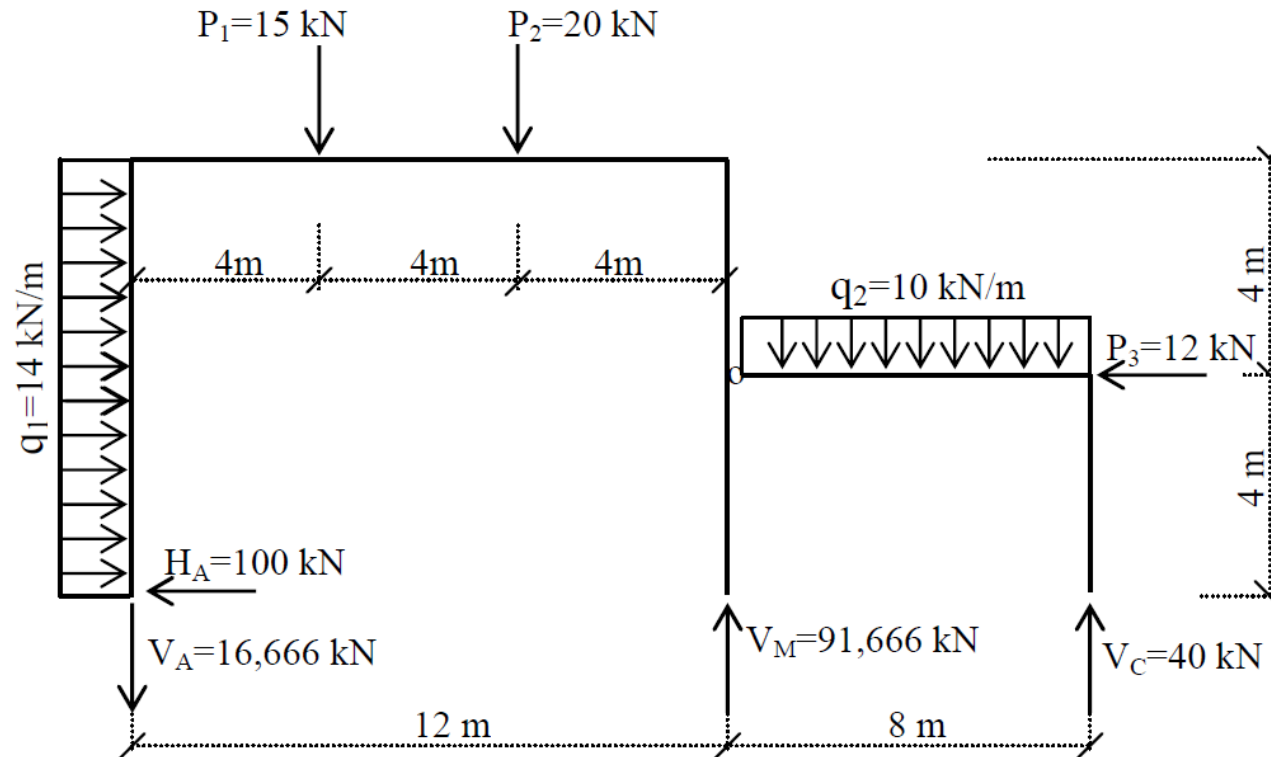
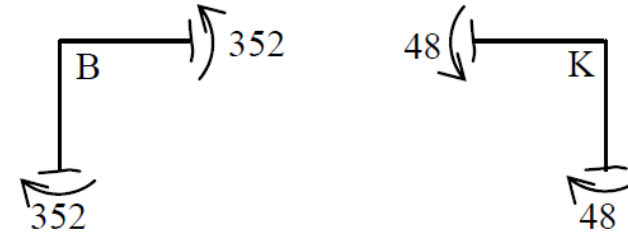
$$\sum M_K = 48 - 48 = 0$$



Kontrol :

$$\sum M_B = 352 - 352 = 0$$

$$\sum M_K = 48 - 48 = 0$$



$$\sum X = q_1 \cdot 8 - H_A - P_3 = 14 \cdot 8 - 100 - 12 = 0$$

$$\sum Y = -V_A - P_1 - P_2 + V_M - q_2 \cdot 8 + V_C = 16,666 - 15 - 20 + 91,666 - 10 \cdot 8 + 40 = 0$$